行政院國家科學委員會補助專題研究計畫 ■ 成 果 報 告
□期中進度報告

# 移動型機器手臂之非線性強健控制

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## 中、英文摘要及關鍵詞

本計劃旨在對移動型機器手臂之非線性強健控制研究，由於輪型移動平台受限於 nonholonomic 條件，對某些控制設計方法將造成困難。經過研究，以適應控制之速度增益法，較為人所常用；此法不但能使系統穩定，且也能滿足軌跡追蹤及速度追隨之性能要求。
關鍵詞：非線性強健控制、移動型機器手臂、nonholonomic 條件、速度增益法、軌跡追蹤、速度追隨

In this project, a study of the nonlinear robust control for a mobile manipulator is proposed. Due to the wheeled mobile is constrained to some nonholonomic conditions, a difficulty will be occurred for some control design technique. After comparison, it is preferred to use a velocity gain control method to design a controller. By using this method, it is shown a nonlinear mobile system can be stabilized and some performance can also be achieved such as circular trajectory tracking and velocity following.
Key words: nonlinear robust control, mobile manipulator, nonholonomic conditions, velocity gain control, trajectory tracking, velocity following

**The Robust Control of the Mobile Manipulator Controlled by Velocity Gain Method**

**Abstract**
In this project, a study of the nonlinear robust control for a mobile manipulator is proposed. Due to the wheeled mobile is constrained to some nonholonomic conditions, a difficulty will be occurred for some control design technique. After comparison, it is preferred to use a velocity gain control method to design a controller. By using this method, it is shown a nonlinear mobile system can be stabilized and some performance can also be achieved such as circular trajectory tracking and velocity following.

## 1. Introduction

As a role of the robot plays is more versatile in industry and in human life, the study of the dynamic behaviors of the robot increasingly attract many researcher's attention in the last decade. Most start a basic model, mobile manipulator, which is composed of a manipulator and mobile platform. Due to the dynamic behavior of the wheeled platform is confined to the nonholonomic constraints, as results the dynamic equations of the platform will be nonlinear and parameter-dependent. How to solve this problem still remains in open issue. Yamamoto [1-3] focused on the dynamic behavior of a moving platform and the manipulator position will be the final position of the mobile platform though the transformation. As defined the output equation is the final position of the manipulator position, the command following problem can be rephrased as the trajectory tracking problem. The mobile platform is to be controlled so that the manipulator can follow the desired trajectories. By using input-output linearization, a control law that transforms the nonlinear equation to the linear equation can be found by Yamamoto [3]. Shen Lin and Goldernberg proposed a neural-network control of mobile manipulator [8-9]. Using neural-network control with on-line learning algorithms, they can design a robust-adaptive controller without requiring off-line tuning. Fierro and Lewis [6] also design a controller by application of neural-network theory. However, they combine adaptive backstepping and Lyapunov stability theory to develop so called a kinematic/torque control law to design a controller. By using this technique, they can deal with more control problems such as: tracking a reference trajectory, path following and stabilization about a desired posture.
problem is made. However, for further solving this problem some difficulties are occurred. As a result, it is preferred for many researchers to deal this problem by using the adapting control law or neural-network control technique. We will also go though those control methods to gain insight the limitation of each technique.
This paper is organized as follows. Section 2 is the dynamic model of the wheeled mobile manipulator. To address the mobile manipulator problem as a nonlinear $H_\infty$ problem and discuss some difficulties for solving the problem is in section 3. Section 4 presents an adaptive control design to solve the nonlinear mobile manipulator problem. Simulation result is presented in section 5. Final remark is in section 6.

## 2. The Dynamic Model of a wheeled Mobile Manipulator

The dynamic model of a wheeled mobile platform, which is constrained to some Nonholonomic conditions, can be described as the following:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(q,\dot{q}) = E(q)\tau - A^T(q)\lambda \qquad (1)$$

In which $q \in \Re^n$ is the generalized coordinates, $M(q)$ is $n \times n$ inertial matrix, $C(q,\dot{q})$ is $\Re^{n\times n}$ the centripetal and coriolis matrix, $F(q,\dot{q}) \in \Re^n$ is the friction and gravity matrix, $E(q)$ is $n \times r$ input vector, $\tau$ is $r$ dimensional input vector, $A(q)$ is $m \times n$ Jacobin matrix and $\lambda$ is the vector constrain force.

The $m$ kinematic constrains is defined as

$$A(q)\dot{q} = 0 \tag{2}$$

Let $s_1(q),.....,s_{n-m}(q)$ are a set of smooth and linearly independent matrices which is the null space of $A(q)$

$$A(q)s_i(q) = 0 \qquad i = 1,......,n-m \tag{3}$$

Let $S(q)$ is collected by this vector and constituted a full rank matrix.

$$S(q) = [s_1(q) \ldots s_{n-m}(q)] \tag{4}$$

Then we can define a steering system as follows

$$\dot{q} = S(q)V(t) \tag{5}$$

$V(t) \in \Re^{n-m}$ denotes a velocity input vector that drives the state vector $q(t)$ into the state space

In this project, an attempt of the formulating the wheeled mobile manipulator as a nonlinear $H_\infty$

For simplifying, the structure of the mobile platform is given as the following:

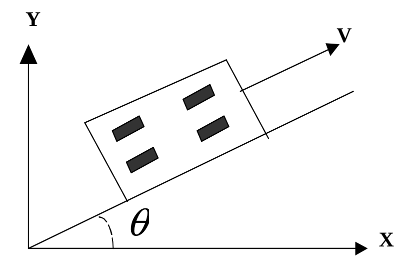



Figure 1. The coordinate of the mobile platform

$q = (x, y, \theta)^T$ . In which $(x, y)$ is the Cartesian coordinate of the center of mass (COM) of the mobile platform and $\theta$ is the orientation of the platform.

$$M(q) = \begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & md\dot{\theta}^2\cos\theta \\ 0 & 0 & md\theta^2 \\ 0 & 0 & 0 \end{bmatrix}$$

$m$ : the mass of the platform

$d$ : the distance between COM and the center axis of the wheel.

$I$ : the moment of the inertial.

$$E(q)=\frac{1}{\rho}\begin{bmatrix}\cos\theta & \cos\theta\\ \sin\theta & \sin\theta\\ R & -R\end{bmatrix}$$

$\lambda=-m(\dot{x}\cos\theta+\dot{y}\sin\theta)\dot{\theta}$

$\rho$ : the radius of the wheel.

$R$ : the distance from the center axis to the wheel.

$$A(q)^T=\begin{bmatrix}-\sin\theta\\ \cos\theta\\ -d\end{bmatrix} \qquad S(q)=\begin{bmatrix}\cos\theta & -d\sin\theta\\ \sin\theta & d\cos\theta\\ 0 & 1\end{bmatrix}$$

The steering system in (5) can be expressed as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & -d\sin\theta\\ \sin\theta & d\cos\theta\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}v_1\\ v_2\end{bmatrix} \tag{6}$$

In which $v_1$ represents the forward linear velocity and $v_2$ is the angular velocity. Multiplying (1) by $S^T$ in both sides, we have the following:

$$S^T MS\dot{V}+S^T(M\dot{S}+CS)V+S^T F=S^T E\tau \tag{7}$$

or simplified as

$$\overline{M}\dot{V}+\overline{C}V+\overline{F}=\overline{\tau} \tag{8}$$

where $\overline{M}=S^T MS$ , $\overline{C}=S^T(M\dot{S}+CS)$ , $\overline{F}=S^T F$ , $\overline{\tau}=S^T E\tau$

So, the dynamic of the mobile platform is in (8) and the kinmatic of the system is described in (6).

**3. A difficulty to solve the mobile platform formulated as a nonlinear $H_\infty$ control problem**

The nonlinear system of the vehicle $\tilde{P}$ according to (8) is the following:

The nonlinear system $\tilde{P}$

Dynamics: $\dot{V}=(\overline{M})^{-1}\overline{\tau}-(\overline{M})^{-1}\overline{C}V-(\overline{M})^{-1}\overline{F}$ . (9)

For simplifying, assume $\overline{F}=0, V=\begin{bmatrix}v_1\\ v_2\end{bmatrix}$ and $\tau=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$

Then (9) will be

$$\begin{bmatrix}\dot{v}_1\\ \dot{v}_2\end{bmatrix}=\begin{bmatrix}0 & d\dot{\theta}[1-\dfrac{\dot{\theta}(2\sin\theta+\sin 2\theta)}{2}]\\ 0 & \dfrac{d^2m(\cos 2\theta+2\cos\theta-1)}{2d^2m-2}\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}+\begin{bmatrix}\dfrac{1}{m\rho} & \dfrac{1}{m\rho}\\ \dfrac{R}{\rho-d^2m\rho} & \dfrac{R}{-\rho+d^2m\rho}\end{bmatrix}\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \tag{10}$$

The plant in (10) is a parameter dependent ($\dot{\theta},\theta$) and nonlinear system. It is difficulty to use a nonlinear $H_\infty$ control technique to design a controller to solve this problem. The reason has been explained by Jean-Jacques Slotine and Weipiing Li in [16]. The most nonlinear $H_\infty$ control problem is to deal with a constant or slowly-varying parameters and a priori estimates of the parameter bounds should be required before-hand.

**4. The adaptive control design**

In the adaptive control, we will discuss the issue of stabilizing and performance.
Consider a reference cart is defined as the following

$$\dot{q}_r = S(q_r)V_r$$

where

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\theta}_r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -d\sin\theta_r \\ \sin\theta_r & d\cos\theta_r \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix} \tag{11}$$

The tracking errors with respect to the mobile platform are defined as follow:

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{12}$$

Differentiating (12) and plugging (6) and (11), we have

$$\dot{e} = \begin{bmatrix} v_r \cos e_3 + v_2 e_2 - v_1 \\ v_r \sin e_3 - v_2 e_1 \\ w_r - v_2 \end{bmatrix} \tag{13}$$

Theorem [8]: Providing the following velocity control law

$V_c = \begin{bmatrix} v_{1c} \\ v_{2c} \end{bmatrix}$ defined as

$$v_{1c} = k_1 e_1 + v_r \cos e_3$$

$$v_{2c} = w_r + e_2 v_r + k_2 v_r \sin e_3 \tag{14}$$

and $k_1 > 0, k_2 > 0$ are gains to be adjusted.

Proof:

The proof is simple and a proposed Lyapunov function is defined as

$$H(e) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + (1 - \cos e_3) \tag{15}$$

If the proposed control law can satisfy the following inequality

$$\dot{H}(e) < 0$$

then the nonlinear system can be stabilized.

$$\dot{H}(e) = e_1\dot{e}_1 + e_2\dot{e}_2 + \dot{e}_3 \sin e_3 \tag{16}$$

Plugging (13) into (16), we can have

$$\dot{H}(e) = e_1(v_r \cos e_3 + v_2 e_2 - v_1) + e_2(v_r \sin e_3 - v_2 e_1) + \sin e_3 (w_r - v_2) \tag{17}$$

replacing the control law (14) into $(v_1, v_2)$, so it yields as

$$\dot{H}(e) = e_1(-k_1 e_1) - \sin e_3 (k_2 v_r \sin e_3) = -e_1^2 k_1 - k_2 v_r \sin^2 e_3 \qquad (18)$$

Since $k_1, k_2$ are positive and $v_r$ is reference velocity chosen as positive, then $\dot{H}(e) < 0$

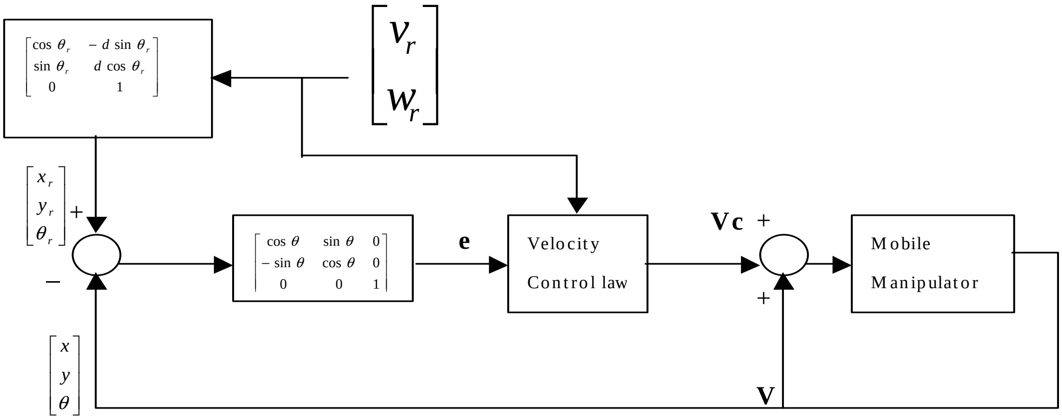



Figure 2. The closed loop system of the mobile manipulator using the velocity control law.

**5. Simulation**

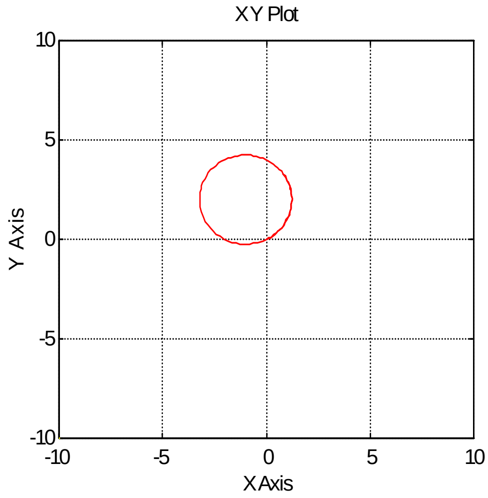
The simulation result is based on the following example:

Given the reference input $V_r = \begin{bmatrix} v_r \\ w_r \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$ and the initial conditions was set to zero

$$\begin{bmatrix} x_r(0) \\ y_r(0) \\ \theta_r(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The property of the mobile cart is the following: $m = 10$, $I = 5$ and $\rho = 2$.

Considering the reference cart is in a circular path shown in figure 3 with the velocity $V_r = [2.5 \quad 1]^T$ and the simulation results presented in figure 4 and figure 5. It is shown that the velocity control law can achieve the performance requirement.



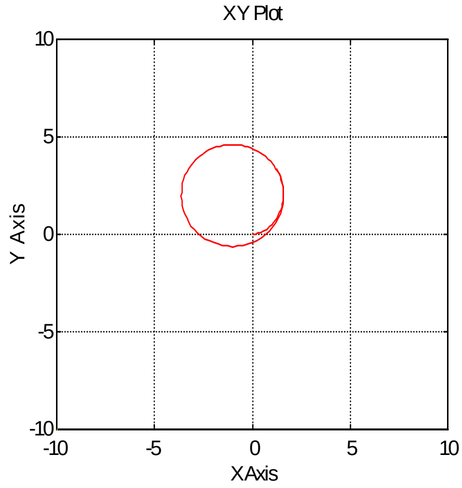




Figure 3. The reference cart in a circular path

Figure 4. The mobile manipulator tracking the circular path

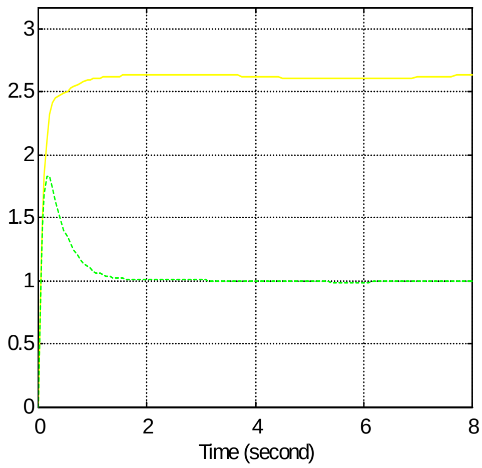



Figure 5. The mobile manipulator following the reference velocity using the velocity control law

**6.The Conclusion**

In this project, the study of the mobile manipulator by employing the nonlinear $H_\infty$ control problem and the adaptive control method are proposed. In achieving some performance such stabilized a system , tracking path and velocity following, it is preferred to use velocity control law than the nonlinear $H_\infty$ control method when dealing with a problem with parameter-dependent system.

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**計劃成果自評：**

本計劃係對輪型移動機器手臂進行控制器設計，首先進行動力分析，由於輪型移動之動力行為，受限於 nonholonmic 條件；車身前進方向與歸向角度(heading angle)有關，此一 coupling effect 增加了推導運動方程式的困難。目前的推導，都是先從 kinematics 著手，由位置座標與前進方位之轉換矩陣，找出零空間(null space)，再由零空間對運動方程式作轉換，及得到滿足 nonholonmic 條件之運動方程式，接下來的工作就是控制器設計。

然而，經轉換推導所得之運動方程式，為一非線性且為參數相依(parameter dependent)之運動方程式，本計劃研究之目的在於使用所用非線性強健控制方法，設計控制器，除了穩定輪型移動機器手臂系統外，尚須要求滿足軌跡追蹤及速度追隨性能要求。首先以傳統非線性 $H_\infty$ 控制法則設計控制器，將穩定、尋跡及速度追隨，當作控制輸入並結合原系統之參考輸入，而構建一個廣義之受控體(generalized plant)，對於原系統之參數相依項，是否可以視作系統之變異量(perturbation)，藉由構建廣義受控體時，看成為 $P-\Delta$ structure，而把參數相依項獨立出去？但這面臨了兩個困難：一、對於一個非線性且參數相依之系統，此一參數項非以代數形式(adding or multiply)存在於系統，而是二次式(square)形式以上存在系統方程式。再者就非線性而言；非線性 $H_\infty$ 問題，其解存在之先決條件，便是非線性方程式線性化後，其 $H_\infty$ 解存在。以輪型移動機器手臂所推導之非線性參數相依運動方程式，其線性化後之 $H_\infty$ 解不存在。二、就物理意義(physic sense)而言，此一參數其實便是歸向角，其大小會對其他方向之位移或速度有 coupling effect。是無法視作 $P-\Delta$ structure 能單獨獨立出去。

以適應控制來設計強健性控器並非新創；目前做法，從一個非線性參數相依運動方程式，先定一個參考運動方程式，強健性問題可轉換成穩定、尋跡及速度追隨問題，與參考運動量可先定義出誤差量(error)及誤差量之微分，然後定義與誤差量和其微分有關之李雅普諾夫函數 (Lyapunov function) ，在滿足李雅普諾夫函數時間變量必須為負之要求下，可設計控制器。經推導以速度增益法較其他方法來得簡單。

經由 ITI 、Mathematic 及 Matlab 軟體模擬圓周軌跡追蹤及速度追隨，此法證明系統可被穩定並滿足尋跡及速度追隨等性能。

全案完成輪型移動機器手臂控制器設計，達成預期目標。

本篇研究可在相關期刊論文發表