行政院國家科學委員會專題研究計畫成果報告

以 GDQ 法做壓電材料的數值分析

Numerical Analyses of Piezoelectric Materials with the GDQ Method

計畫編號：NSC 90-2212-E-164-005

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一、中文摘要

我們採用 GDQ 數值方法[4~7]，用沒有 Overshoot 和沒有 Gibbs 影響的上、下表面之對稱壓力，做 PZT-5H 壓電材料受到上、下表面之電壓作用及兩自由端的研究分析，得到應力$\pm_1$，$\pm_3$和$\pm_5$分別對x_1和x_3的應力分佈圖，以及電位函數 ¨ 之分佈圖。我們得知在壓電材料受到壓力和電壓作用的研究分析中，二維的 GDQ 數值方法可以提供一非常有效率的數值解。

關鍵詞：壓電材料、電力負載、一般化微分級數和、電位函數、應力。

Abstract

The two-dimensional generalized differential quadrature (GDQ) method [4~7] was used to study a piezoelectric strip PZT-5H with mechanical and electric loads. We apply the discretized equations of boundary condition of stresses and electric potential function directly into the discretized governing equations to compute the stresses $\pm_1$, $\pm_3$, $\pm_5$ and the electric potential function ¨. There were no overshoot and no Gibbs effect for the local symmetric pressures on the upper and lower edges. The GDQ numerical solutions of local symmetric pressure and constant voltages on the upper and lower edges have been obtained for traction-free boundary conditions on both end surfaces. The present GDQ solutions show that it give an efficient method in the study of stresses and electric potential function for the piezoelectric materials.

Keywords: Piezoelectric Material、Electric loads、GDQ、Electric Potential Function、Stress。

二、緣由與目的

Recently, piezoelectric materials have been widely used in the electronic and electromechanical industries. For example, some types of intelligent structure, actuator and transducer have been made. It is interesting to know the basic characteristics of piezoelectric materials. When a pressure is loading on the surface of the piezoelectric materials, the material would occur a corresponding electric charge. When a voltage is applied on the edge of some piezoelectric materials, the material would occur a corresponding deformation [1~3]. Because the piezoelectric materials have the property of brittleness, so we need to understand the distribution of stress and electric potential function when the material is under the applied load.

The purpose of this project is to use the GDQ method to study the distribution of stresses and electric potential function of a piezoelectric strip, under the symmetric pressure, voltage on the upper and lower edges, and the traction-free boundary condition. The governing partial differential equations in terms of stress and electric potential function are non-dimensionalized, then we derive these differential equations into the form of series equations by using the GDQ method [8].

三、內容

After substituting the discretized equations of boundary conditions into the discretized equations of governing equations, we have the discretized equations of piezoelectric strip for the stresses and the electric potential function in the grid point (i, j) as follows:

$$\frac{(2s_{13}+s_{55})q_0}{\ell^{*2}}\sum_{l=2}^{N-1}A_{i,l}^{(2)}\bar{\pm}_{1_{l,j}}+\frac{s_{11}q_0}{4h^2}\sum_{m=2}^{M-1}C_{3A}\bar{\pm}_{1_{i,m}}$$
$$+\frac{s_{33}q_0}{\ell^{*2}}\sum_{l=2}^{N-1}A_{i,l}^{(2)}\bar{\pm}_{3_{l,j}}+\frac{s_{11}q_0d_{33}}{4h^2d_{31}}\sum_{m=2}^{M-1}C_{3C}\bar{\pm}_{3_{i,m}}$$
$$-\frac{d_{31}V_1}{8h^3}\sum_{m=2}^{M-1}B_{j,m}^{(3)}\bar{\ddot{\ }}_{i,m}=F_1$$

$$\frac{q_0}{2h}\sum_{m=2}^{M-1}B_{j,m}^{(1)}\bar{\pm}_{3_{i,m}}+\frac{q_0}{l^*}\sum_{l=2}^{N-1}A_{i,l}^{(1)}\bar{\pm}_{5_{l,j}}=F_2$$

$$\frac{q_0}{\ell^*}\sum_{l=2}^{N-1}A_{i,l}^{(1)}\bar{\pm}_{1_{l,j}}+\frac{q_0}{2h}\sum_{m=2}^{M-1}B_{j,m}^{(1)}\bar{\pm}_{5_{i,m}}=F_3$$

$$\frac{d_{31}q_0}{2h}\sum_{m=2}^{M-1}C_{3B}\bar{\pm}_{1_{i,m}}+\frac{(d_{33}-d_{15})q_0}{2h}\sum_{m=2}^{M-1}B_{j,m}^{(1)}\bar{\pm}_{3_{i,m}}$$
$$+\frac{d_{31}q_0}{2h}\sum_{m=2}^{M-1}C_{3D}\bar{\pm}_{3_{i,m}}$$
$$-\frac{\pounds_{33}V_1}{4h^2}\sum_{m=2}^{M-1}B_{j,m}^{(2)}\bar{\ddot{\ }}_{i,m}=F_4$$

Where

$C_{3A}=B_{j,m}^{(2)}+C_{3C}$,

$C_{3B}=B_{j,m}^{(1)}+C_{3D}$,

$C_{3C}=B_{j,1}^{(2)}C_{3C1}+B_{j,M}^{(2)}C_{3C2}$

$C_{3D}=B_{j,1}^{(1)}C_{3C1}+B_{j,M}^{(1)}C_{3C2}$,

$$C_{3C1}=(-B_{M,M}^{(1)}B_{1,m}^{(1)}+B_{1,M}^{(1)}B_{M,m}^{(1)})/(B_{1,1}^{(1)}B_{M,M}^{(1)}-B_{1,M}^{(1)}B_{M,1}^{(1)})$$

$$C_{3C2}=(B_{M,1}^{(1)}B_{1,m}^{(1)}-B_{1,1}^{(1)}B_{M,m}^{(1)})/(B_{1,1}^{(1)}B_{M,M}^{(1)}-B_{1,M}^{(1)}B_{M,1}^{(1)})$$

$$F_1=\frac{d_{31}V_1}{8h^3}(B_{j,1}^{(3)}\bar{\ddot{\ }}_{i,1}+B_{j,M}^{(3)}\bar{\ddot{\ }}_{i,M})$$
$$+\frac{s_{11}q_0d_{33}}{4h^2d_{31}}(B_{j,1}^{(2)}\bar{\pm}_{3_{i,1}}+B_{j,M}^{(2)}\bar{\pm}_{3_{i,M}})$$

$$F_2=\frac{-q_0}{2h}(B_{j,1}^{(1)}\bar{\pm}_{3_{i,1}}+B_{j,M}^{(1)}\bar{\pm}_{3_{i,M}})$$

$F_3=0$

$$F_4=-\frac{(d_{33}-d_{15})q_0}{2h}(B_{j,1}^{(1)}\bar{\pm}_{3_{i,1}}+B_{j,M}^{(1)}\bar{\pm}_{3_{i,M}})$$
$$+\frac{\pounds_{33}V_1}{4h^2}(B_{j,1}^{(2)}\bar{\ddot{\ }}_{i,1}+B_{j,M}^{(2)}\bar{\ddot{\ }}_{i,M})$$
$$+\frac{q_0d_{33}}{2h}(B_{j,1}^{(1)}\bar{\pm}_{3_{i,1}}+B_{j,M}^{(1)}\bar{\pm}_{3_{i,M}})$$

四、結果與討論

We consider the PZT-5H piezoceramic strip with the following material property constants:

$s_{11}=16.5\times10^{-12}\,m^2/N$

$s_{13}=-8.45\times10^{-12}\,m^2/N$

$s_{33}=20.7\times10^{-12}\,m^2/N$

$s_{55}=43.5\times10^{-12}\,m^2/N$

$d_{31}=-274\times10^{-12}\,C/N$

$d_{33}=593\times10^{-12}\,C/N$

$d_{15}=741\times10^{-12}\,C/N$

$\pounds_0=8.85\times10^{-12}\,F/m$

$\pounds_{11}/\pounds_0=1700$

$\pounds_{33}/\pounds_0=1470$

And with the geometric values: $\ell^*=10mm$, $h=1mm$, under the pressure load $q_0=20{,}000{,}000N/m^2=20MPa$, $2a=5mm$, electric voltage $V_1=1000V$, $V_0=0V$. For the numerical computation, we let the computational domain $0\le X\le1$, $0\le Y\le1$ be divided into $N-1$ intervals with coordinates as X_1, $X_2,...,X_N$ and $M-1$ intervals with coordinates as Y_1, $Y_2,...,Y_M$, respectively. Then we have the $N\times M$ grid points for a piezoelectric strip and have the typical grid point (i,j) , $i=1,2,\ldots,N$, $j=1,2,\ldots,M$

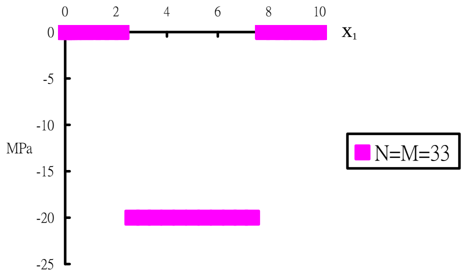

Figure 1 Typical pressure load q_0 of grids 33×33.

Figure 1 show that the typical pressure load q_0 of grids 33×33. There were no overshoot and no Gibbs effect for the local symmetric pressures on the upper and lower edges.

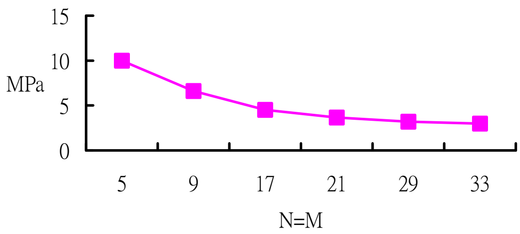

Figure 2 $\pm_1(\frac{\ell^*}{2}, h)$ convergence v.s N=M

Figure 2 show that the convergence of stress component $\pm_1(\frac{\ell^*}{2}, h)$ with respective to grids at the center position of the strip. We then use the grids of $N = M = 33 \times 33$ to make the numerical GDQ solution of stress distribution and electric potential function.

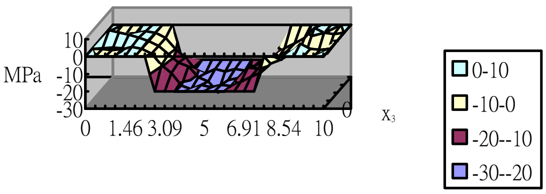

Figure 3 $\pm_1$ (MPa) distribution with GDQ method.

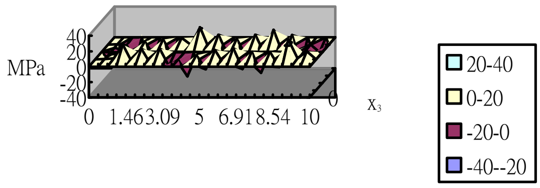

Figure 4 $\pm_3$ (MPa) distribution with GDQ method

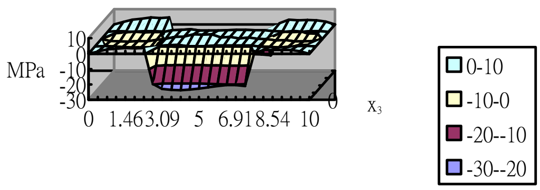

Figure 5 $\pm_5$ (MPa) distribution with GDQ method

Figure 3-5 show that $\pm_1$, $\pm_3$ and $\pm_5$ distributions with respective to x_1 and x_3 by using the GDQ method of grids 33×33.

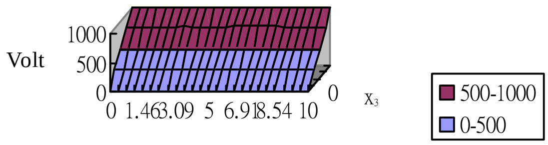

Figure 6 ¨ (Volt) distribution with GDQ method

Figure 6 show that the linear electrical potential ¨ distribution with GDQ method of grids 33×33. There are the coupling results between the stress and electric fields. We find that the $\pm_1$ distribution has very large value within the region of pressure load along the lower and upper surfaces of the piezoceramic strip.

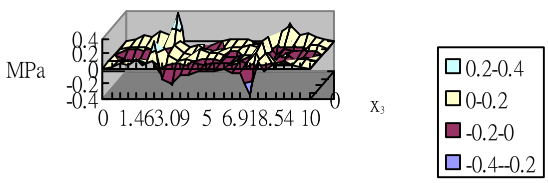

Figure 7 $\pm_1$ (MPa) distribution with GDQ method of grids 33×33 without piezoelectric effect.

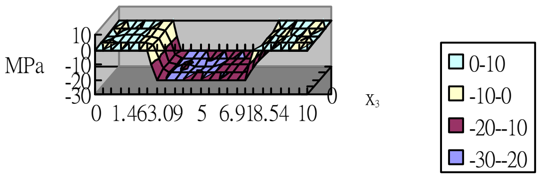

Figure 8 $\pm_3$ (MPa) distribution with GDQ method of grids 33×33 without piezoelectric effect.

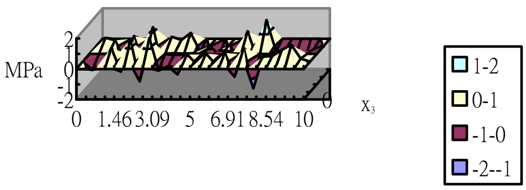

Figure 9 $\pm_5$ (MPa) distribution with GDQ method of grids 33×33 without piezoelectric effect.

When we only consider the elastic characteristics of material without the effects of piezoelectric characteristics and voltages ($d_{ij} = 0$, $£_{ij} = 0$, $V_1 = 0$), the corresponding stress distribution $\pm_1$, $\pm_3$ and $\pm_5$ with respective to x_1 and x_3 by using the GDQ method of grids 33×33 are shown in Figure 7-9. We find that stress distribution $\pm_1$, $\pm_3$ and $\pm_5$ with respective to x_1 and x_3 are strongly influence by the piezoelectric characteristics and voltages.

五、計畫成果自評

The two-dimensional generalized differential quadrature (GDQ) method provided an efficient solution to study a piezoelectric strip with the coupling effect of mechanical and electric loads.

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共同主持人：

計畫參與人員：

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中　華　民　國　九十一　年　七　月　十六　日