GDQ

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GDQ

(Computer Approach of Laminated Piezoelectric Shells by the GDQ Method)

GDQ

3 $[90^{o}/0^{o}/90^{o}]$ PVDF

$(FSDT)$ GDQ

$\overline{W}$ $\overline{\sigma}_x$ $\overline{\sigma}_\theta$

Abstract

A piezoelectric laminated cylindrical shell with shear rotations effect under the electromechanical loads and four sides simply supported boundary condition was studied by using the two-dimensional generalized differential quadrature (GDQ) computational method. The typical hybrid composite shells with 3-layered cross-ply $[90^{o}/0^{o}/90^{o}]$ graphite-epoxy laminate and bounded PVDF layers are considered under the sinusoidal pressure loads and electric potentials on the shell. The governing partial differential equation with First-order Shear Deformation Theory (FSDT) in terms of mid-surface displacements and shear rotations can be expressed in series equations by the GDQ formulation. Thus we obtain the GDQ numerical solutions of displacement ratio $\overline{W}$, stress ratios $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$.

Keywords: generalized differential quadrature (GDQ), piezoelectric laminated cylindrical shell, displacement ratio, stress ratios.

1.

Kapuria, Sengupta 和 Dumir [6-7] 在 1998

Navier-type 在 1997 年 Exact Analytical Fourier Series

1998 年 Mirfakhraei 和 Redekop 用 Differential Quadrature Method (DQM)

[8] (Generalized
Differential Quadrature ,GDQ)

2.

GDQ

3.

2002 Liu 和 Fan 用边界元法(Boundary Element Method, BEM)

[1] Jun 和 Zhaowei 用有限元法(Finite Element Method,
FEM) PZT [2] Dubus, Haw, Granger 和 Ledez
[3] 2000 Benjeddou

adaptive [4] 1998 Khutoryansky, Sosa 和 Zu
active [5] :
2000 GDQ [17-19]
[20] 1999-2001
[21-24] 2003 GDQ
[25-26]

4.

Basically, the GDQ method can be stated that: the derivative of a smooth function at a discrete
point in a domain can be discretized by using an approximated weighting linear sum of the function
values at all the discrete points in the direction [9-16].

We use the following non-dimensionalized parameter: $X = \dfrac{x}{a}$, $Y = \dfrac{\theta}{\psi}$, $U = \dfrac{u^0}{a}$, $V = \dfrac{u^0}{h^*}$,

$W = \dfrac{w^0}{h^*}$. And after substituting the discretized discretized equations of governing equations, we

have the discretized equations of piezoelectric shell for the stresses and the electric potential
function in the grid point (i, j) as follows:

$$\frac{(A_{11} + \overline{B}_{11})}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} U_{\lambda,j} + \frac{a(A_{66} - \overline{B}_{66} + \overline{D}_{66})/R^2}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} U_{i,m} + \frac{h^*(A_{12} + A_{66})/R}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} V_{\lambda,m}$$

$$+ \frac{h^* A_{12}/R}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} W_{\lambda,j} + \frac{B_{11} + D_{11}/R}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} \phi_{1_{\lambda,j}} + \frac{(\overline{B}_{66} - \overline{D}_{66})/R}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} \phi_{1_{i,m}}$$

$$+ \frac{\overline{B}_{12} + \overline{B}_{66}}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} \phi_{2_{\lambda,m}} = -P_{x_{o,j}} - \frac{1}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} N_{x_{\lambda,j}}^e$$

$$\frac{a(A_{12} + A_{66})/R}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} U_{\lambda,m} + \frac{h^*(A_{66} + \overline{B}_{66})}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} V_{\lambda,j}$$

$$+ \frac{h^*(A_{22} - \overline{B}_{22} + \overline{D}_{22})/R^2}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} V_{i,m} + \frac{h^*(\overline{B}_{44} - \overline{D}_{44} - A_{44})}{R^2} V_{i,j}$$

$$+ \frac{h^*(A_{44} - \overline{B}_{44} + \overline{D}_{44} + A_{22} - \overline{B}_{22} + \overline{D}_{22})/R^2}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} W_{i,m} + \frac{(\overline{B}_{12} + \overline{B}_{66})}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} \phi_{1_{\lambda,m}}$$

1

$$+ \frac{B_{66} + D_{66}/R}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} \phi_{2_{\lambda,j}} + \frac{(\overline{B}_{22} - \overline{D}_{22})/R}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} \phi_{2_{i,m}} + \frac{\overline{B}_{44} - \overline{D}_{44} - A_{44}}{-R^2} \phi_{2_{i,j}}$$

$$= -P_{\theta_{i,j}} - \frac{1}{R}(Q_{\theta_{i,j}}^e + \frac{1}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} N_{\theta_{i,m}}^e)$$

$$\frac{aA_{12}/R}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} U_{\lambda,j} + \frac{h^*(A_{44} - \overline{B}_{44} + \overline{D}_{44} + A_{22} - \overline{B}_{22} + \overline{D}_{22})/R^2}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} V_{i,m}$$

$$+ \frac{h^*(-A_{55} - \overline{B}_{55})}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} W_{\lambda,j} + \frac{h^*(\overline{B}_{44} - \overline{D}_{44} - A_{44})/R^2}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} W_{i,m}$$

$$+ h^*(A_{22} - \overline{B}_{22} + \overline{D}_{22})/R^2 W_{i,j} + \frac{(\overline{B}_{12} - \overline{B}_{55} - A_{55})}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \phi_{1_{\lambda,j}}$$

$$+ \frac{(\overline{B}_{22} + \overline{B}_{44} - A_{44} - \overline{D}_{22} - \overline{D}_{44})/R}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} \phi_{2_{i,m}} = P_{z_{i,j}} + \frac{1}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} Q_{x_{\lambda,j}}^e + \frac{1}{R}(\frac{1}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} Q_{\theta_{i,m}}^e - N_{\theta_{i,j}}^e)$$

$$\frac{(B_{11} + D_{11}/R)}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} U_{\lambda,j} + \frac{a(\overline{B}_{66} - \overline{D}_{66})/R}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} U_{i,m} + \frac{h^*(\overline{B}_{12} + \overline{B}_{66})}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} V_{\lambda,m}$$

$$+ \frac{h^*(\overline{B}_{12} - \overline{B}_{55} - A_{55})}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} W_{\lambda,j} + \frac{D_{11}}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} \phi_{1_{\lambda,j}} + \frac{\overline{D}_{66}}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} \phi_{1_{i,m}} + (-A_{55} - \overline{B}_{55})\phi_{1_{i,j}}$$

$$+ \frac{(-A_{55} - \overline{B}_{55})}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} \phi_{2_{\lambda,m}} = -m_{x_{i,j}} + Q_{x_{i,j}}^e - \frac{1}{a} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} M_{x_{\lambda,j}}^e$$

$$\frac{a(\overline{B}_{12} + \overline{B}_{66})}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} U_{\lambda,m} + \frac{h^*(B_{66} + D_{66}/R)}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} V_{\lambda,j} + \frac{h^*(\overline{B}_{22} - \overline{D}_{22})/R}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} V_{i,m}$$

$$+ \frac{h^*(\overline{B}_{44} - \overline{D}_{44} - A_{44})}{-R} V_{i,j} + \frac{(\overline{B}_{22} + \overline{B}_{44} - A_{44} - \overline{D}_{22} - \overline{D}_{44})/R}{\psi} \sum_{m=1}^{M} B_{j,m}^{(1)} W_{i,m}$$

$$+ \frac{(-A_{55} - \overline{B}_{55})}{a\psi} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(1)} \sum_{m=1}^{M} B_{j,m}^{(1)} \phi_{1_{\lambda,m}} + \frac{D_{66}}{a^2} \sum_{\lambda=1}^{N} A_{i,\lambda}^{(2)} \phi_{2_{\lambda,j}} + \frac{\overline{D}_{22}}{\psi^2} \sum_{m=1}^{M} B_{j,m}^{(2)} \phi_{2_{i,m}} + (\overline{B}_{44} - \overline{D}_{44} - A_{44})\phi_{2_{i,j}}$$

$$= -m_{\theta_{i,j}} + Q_{\theta_{i,j}}^e - \frac{1}{\psi R} \sum_{m=1}^{M} B_{j,m}^{(1)} M_{\theta_{i,m}}^e$$

And the terms of the contribution to force and moment resultants due to applied electric field $E = [E_x, E_\theta, E_z]^T$ are given for the N_k layers. $E_x = -\phi_{,x}$, $E_\theta = -\phi_{,\theta}/(R+Z)$, $E_z = -\phi_{,z}$ where ϕ_1 and ϕ_2 are the electric potentials at the inner and outer surfaces of the shell. The potentials ϕ are assumed to vary linearly across the layers of shell, $\phi = \phi_1 + z\phi_2$.

5.

(GDQ)

$\overline{W}$ $\overline{\sigma}_x$

$\overline{\sigma}_\theta$ - -

In order to obtain some numerical GDQ results in personal computer, we consider the hybrid composite shell made of cross-ply graphite-epoxy laminate and PVDF (polyvinyledene fluoride) layer bonded to its surface. The typical hybrid composite shell is listed as follows:

Type1: 3-layered cross-ply graphite-epoxy laminate $[90^o/0^o/90^o]$ and an outer PVDF layer.

Type2: 3-layered cross-ply graphite-epoxy laminate $[90^o/0^o/90^o]$ and PVDF layers bonded to its inner and outer surfaces.

The material properties of graphite-epoxy are given as follows [6]:

$E_L = 172.5\,GPa$, $E_T = 6.9\,GPa$, $G_{LT} = 3.45\,GPa$, $G_{TT} = 1.38\,GPa$, $\gamma_{LT} = \gamma_{TT} = 0.25$, $d_i = 0$.

And the material properties of PVDF are given as follows[6]:

$E_i = 2.0\,GPa$, $\gamma_{ij} = \dfrac{1}{3}$, $d_1 = 3 \times 10^{-12}\,CN^{-1}$, $d_2 = 23 \times 10^{-12}\,CN^{-1}$, $d_3 = -30 \times 10^{-12}\,CN^{-1}$, $d_4 = 0$,

$d_5 = 0$.

We consider the typical hybrid shell (see Figure1) including shear deformation under four sides simply supported for the following electromechanical loads cases with $P_x = P_\theta = m_x = 0$:

case1: $P_z = -P_0(1 + \dfrac{h^*}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $\phi_1 = \phi_2 = 0$

case2: $P_z = 0$, $\phi_1 = 0$, $\phi_2 = \phi_0 \sin(\dfrac{\pi x}{a})\cos\theta$

And we use the following coordinate of grid points in the GDQ computation:

$x_i = 0.5\left[1 - \cos(\dfrac{i-1}{N-1}\pi)\right]a$, $i = 1,2,\text{K},N$, $\theta_i = 0.5\left[1 - \cos(\dfrac{j-1}{M-1}\pi)\right]\psi$, $j = 1,2,\text{K},M$.

We firstly make the convergence study of deflection W at center position for type1 hybrid shell under case1 load with $P_0 = 1.0MPa$, all plies of hybrid shell have equal thickness, span angle of the panel $\psi = 120^\circ$, length-to-midsurface radius ratio $\dfrac{a}{R} = 1,2,3,4$ and thickness-to-midsurface radius ratio $\dfrac{h^*}{R} = 0.01, 0.05, 0.1, 0.17, 0.25$. The non-dimensional deflection W at center position type1 hybrid shell in GDQ method for the grid point $N \times M = 5 \times 5, 9 \times 9, 13 \times 13, 15 \times 15, 19 \times 19, 23 \times 23, 27 \times 27$ and 31×31 is presented in the Table1. We find that the 31×31 grid point have a good convergence result and can be used further for the hybrid shell including shear deflection analyses under the electromechanical loads and four sides simply supported.

The displacement ratio $\overline{W} = \dfrac{W}{W|_{\frac{h^*}{R}=0.01}}$, and stress ratios $\overline{\sigma}_x = \dfrac{\sigma_x}{\sigma_x|_{\frac{h^*}{R}=0.01}}$, $\overline{\sigma}_\theta = \dfrac{\sigma_\theta}{\sigma_\theta|_{\frac{h^*}{R}=0.01}}$ are defined and used for the following Figures, all the non-dimensional deflection and stress parameters are compared to the corresponding values at $\dfrac{h^*}{R} = 0.01$.

Figure 2 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{h^*}{R}$ at center position of hybrid shell. Type1 under load case1: $P_z = -P_0(1 + \dfrac{h^*}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $P_0 = 1.0MPa$, $\phi_1 = \phi_2 = 0$ and $\dfrac{a}{R} = 4$, $W|_{\frac{h^*}{R}=0.01} = 0.02525$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.04455$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 1.24117$.

We find that $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{h^*}{R}$ increasing, especially in the region $\dfrac{h^*}{R} < 0.05$.

Figure 3 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{a}{R}$ at center position of hybrid shell. Type1 under load case1: $P_z = -P_0(1 + \dfrac{h^*}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $P_0 = 1.0MPa$, $\phi_1 = \phi_2 = 0$ and $\dfrac{h^*}{R} = 0.01$, for the computational value at center position of hybrid shell Type1, $W|_{\frac{h^*}{R}=0.01} = 0.02525$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.04455$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 1.24117$. We find that $\overline{W}$ is almost

constant with $\dfrac{a}{R}$, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{a}{R}$ increasing.

Figure 4 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{h^*}{R}$ at center position of hybrid shell. Type1 under load case2: $p_z = 0$, $\phi_1 = 0, \phi_2 = \phi_0 \sin(\dfrac{\pi x}{a})\cos\theta$ and $\dfrac{a}{R} = 4$ for the computational value at center position of hybrid shell Type2, $W|_{\frac{h^*}{R}=0.01} = 0.390167$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.727941$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 19.1634$. We find that $\overline{W}$ is decreasing with $\dfrac{h^*}{R}$ increasing, but $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are increasing with $\dfrac{h^*}{R}$ increasing.

Figure 5 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{a}{R}$ at center position of hybrid shell. Type1 under load case2: $p_z = 0$, $\phi_1 = 0$, $\phi_2 = \phi_0 \sin(\dfrac{\pi x}{a})\cos\theta$ and $\dfrac{h^*}{R} = 0.01$, for the computational value at center position of hybrid shell Type2, $W|_{\frac{h^*}{R}=0.01} = 0.390167$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.727941$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 19.1634$. We find that $\overline{W}$ is almost constant with $\dfrac{a}{R}$, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{a}{R}$ increasing.

Figure 6 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{h^*}{R}$ at center position of hybrid shell. Type2 under loads case1: $p_z = -p_0(1+\dfrac{h}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $p_0 = 1.0Mp_a, \phi_1 = \phi_2 = 0$ and $\dfrac{h^*}{R} = 0.01$ for the computational value at center position of hybrid shell Type2, $W|_{\frac{h^*}{R}=0.01} = 0.03073$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.4714$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 1.12159$. We find that $\overline{W}$ is almost constant with $\dfrac{h^*}{R}$, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{h^*}{R}$ increasing.

Figure 7 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{a}{R}$ at center position of hybrid shell. Type2 under loads case1: $p_z = -p_0(1+\dfrac{h^*}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $p_0 = 1.0Mp_a, \phi_1 = \phi_2 = 0$ and $\dfrac{h^*}{R} = 0.01$, for the computational value at center position of hybrid shell Type2, $W|_{\frac{h^*}{R}=0.01} = 0.390167$, $\sigma_x|_{\frac{h^*}{R}=0.01} = 0.727941$, $\sigma_\theta|_{\frac{h^*}{R}=0.01} = 19.1634$. We find that $\overline{W}$ is almost constant with $\dfrac{a}{R}$, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{a}{R}$ increasing.

Figure 8 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{h^*}{R}$ at center position of hybrid shell. Type2 under loads case2: $p_z = 0$, $\phi_1 = 0$, $\phi_2 = \phi_0 \sin(\dfrac{\pi x}{a})\cos\theta$ and

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$\dfrac{a}{R} = 4$ for the computational value at center position of hybrid shell Type2, $W\big|_{\frac{h^*}{R}=0.01} = 0.03073$, $\sigma_x\big|_{\frac{h^*}{R}=0.01} = 0.4714$, $\sigma_\theta\big|_{\frac{h^*}{R}=0.01} = 1.12159$. We find that $\overline{W}$ is decreasing with $\dfrac{h^*}{R}$ increasing, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all almost constant with $\dfrac{h^*}{R}$.

Figure 9 shows that the variation of the non-dimensional parameters $\overline{W}, \overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with the $\dfrac{a}{R}$ at center position of hybrid shell. Type2 under loads case2: $p_z = 0$, $\phi_1 = 0$, $\phi_2 = \phi_0 \sin(\dfrac{\pi x}{a})\cos\theta$ and $\dfrac{h^*}{R} = 0.01$ for the computational value at center position of hybrid shell Type2, $W\big|_{\frac{h^*}{R}=0.01} = 0.75958$, $\sigma_x\big|_{\frac{h^*}{R}=0.01} = 1.25817$, $\sigma_\theta\big|_{\frac{h^*}{R}=0.01} = 27.7654$. We find that $\overline{W}$ is almost constant with $\dfrac{a}{R}$, $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ are all decreasing with $\dfrac{a}{R}$ increasing.

Conclusions

The two-dimensional generalized differential quadrature (GDQ) method give us a computational solution of the laminated PVDF hybrid shell under the electromechanical loads and four sides simply supported. We obtain the variations of the non-dimensional parameters of displacement ratio $\overline{W}$, stress ratios $\overline{\sigma}_x$ and $\overline{\sigma}_\theta$ with respect to thickness-to-midsurface radius ratio $\dfrac{h^*}{R}$ and length-to-midsurface radius ratio $\dfrac{a}{R}$ at center position of hybrid shell.

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Table1. Convergence of center deflection W for Type1 Hybrid shell: $[90^\circ/0^\circ/90^\circ]$ and an Outer

PVDF case1 load: $P_z = -P_0(1+\dfrac{h^*}{2R})\sin(\dfrac{\pi x}{a})\cos\theta$, $P_0 = 1.0 MPa$, $\phi_1 = \phi_2 = 0$

| a / R	GDQ Method N×M Grid points	Center deflection W				
		$\dfrac{h^*}{R}=0.01$	$\dfrac{h^*}{R}=0.05$	$\dfrac{h^*}{R}=0.1$	$\dfrac{h^*}{R}=0.17$	$\dfrac{h^*}{R}=0.25$
1	5×5	0.03573	0.00146	0.000381	0.000139	0.0000692
	9×9	0.03468	0.00142	0.000369	0.000135	0.0000673
	13×13	0.03213	0.00132	0.000343	0.000126	0.0000630
	15×15	0.02555	0.00106	0.000278	0.000103	0.0000523
	19×19	0.02693	0.00111	0.000292	0.000108	0.0000544
	23×23	0.02737	0.00113	0.000297	0.000110	0.0000551
	27×27	0.02763	0.00114	0.000298	0.000110	0.0000553
	31×31	0.02753	0.00114	0.000299	0.000110	0.0000553
2	5×5	0.03248	0.00132	0.000339	0.000121	0.0000581
	9×9	0.03161	0.00128	0.000325	0.000117	0.0000564
	13×13	0.02953	0.00120	0.000308	0.000110	0.0000528
	15×15	0.02356	0.00096	0.000264	0.000088	0.0000423
	19×19	0.02495	0.00102	0.000261	0.000093	0.0000448
	23×23	0.02549	0.00104	0.000267	0.000095	0.0000457
	27×27	0.02568	0.00105	0.000269	0.000096	0.0000460
	31×31	0.02581	0.00105	0.000269	0.000096	0.0000461
3	5×5	0.03187	0.00130	0.000332	0.000118	0.0000564
	9×9	0.03104	0.00126	0.000323	0.000115	0.0000547
	13×13	0.02904	0.00118	0.000301	0.000107	0.0000508
	15×15	0.02314	0.00094	0.000241	0.000085	0.0000408
	19×19	0.02459	0.00100	0.000256	0.000091	0.0000434
	23×23	0.02511	0.00102	0.000262	0.000093	0.0000443
	27×27	0.02529	0.00103	0.000264	0.000093	0.0000447
	31×31	0.02539	0.00103	0.000264	0.000094	0.0000448
4	5×5	0.03166	0.00129	0.000330	0.000117	0.0000558
	9×9	0.03085	0.00125	0.000320	0.000114	0.0000543
	13×13	0.02887	0.00117	0.000298	0.000106	0.0000509
	15×15	0.02302	0.00094	0.000239	0.000085	0.0000402
	19×19	0.02446	0.00099	0.000254	0.000090	0.0000430
	23×23	0.02499	0.00102	0.000260	0.000092	0.0000436
	27×27	0.02519	0.00102	0.000262	0.000093	0.0000442
	31×31	0.02525	0.00102	0.000262	0.000093	0.0000444
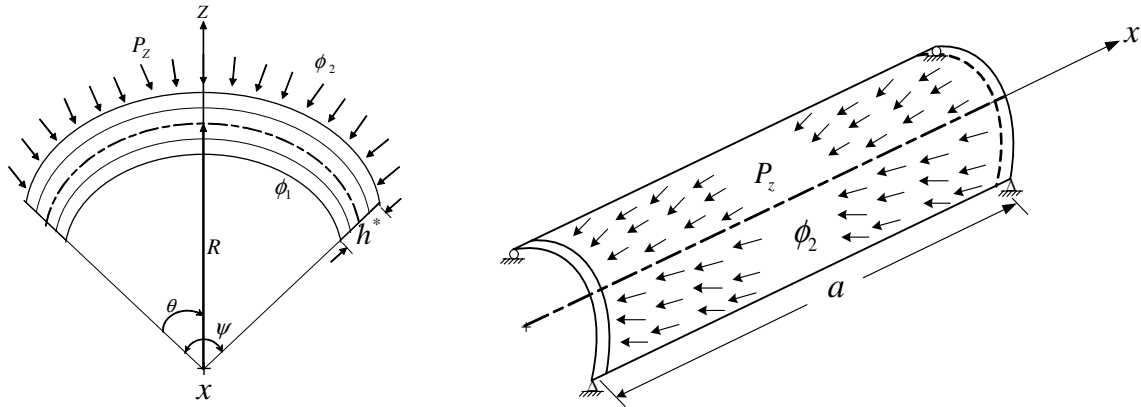
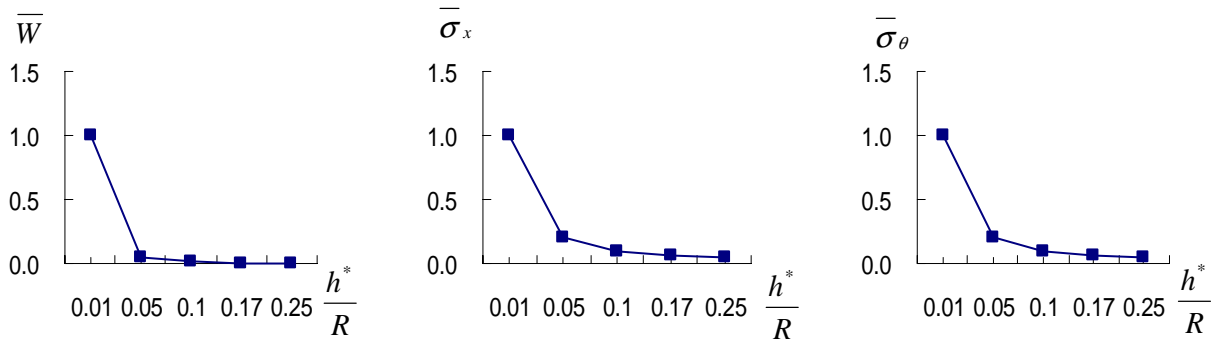Figure 1 Typical geometry of piezoelectric cylindrical shell under pressure and electric loads.

Figure 2 $\dfrac{h^*}{R}$ v.s $\overline{W}, \overline{\sigma}_x, \overline{\sigma}_\theta$ at center position of hybrid shell Type1 under loads case1 and $\dfrac{a}{R} = 4$
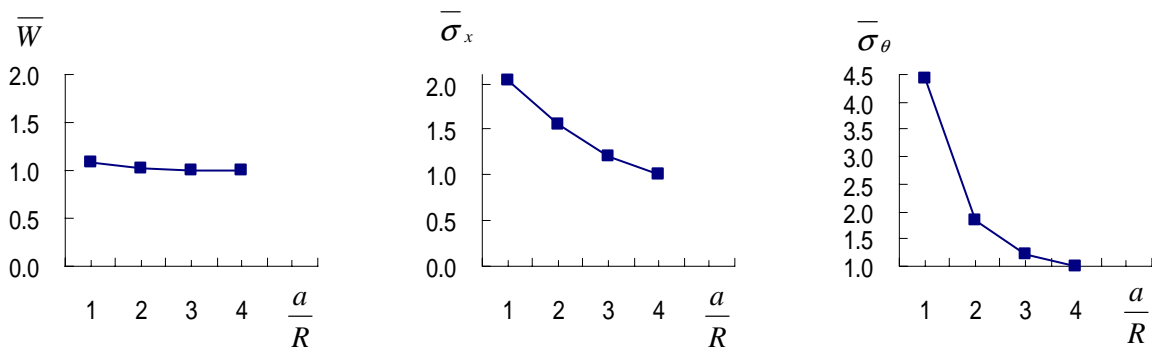

Figure 3 $\dfrac{a}{R}$ v.s $\overline{W}, \overline{\sigma}_x, \overline{\sigma}_\theta$ at center position of hybrid shell Type1 under loads case1 $\dfrac{h^*}{R} = 0.01$
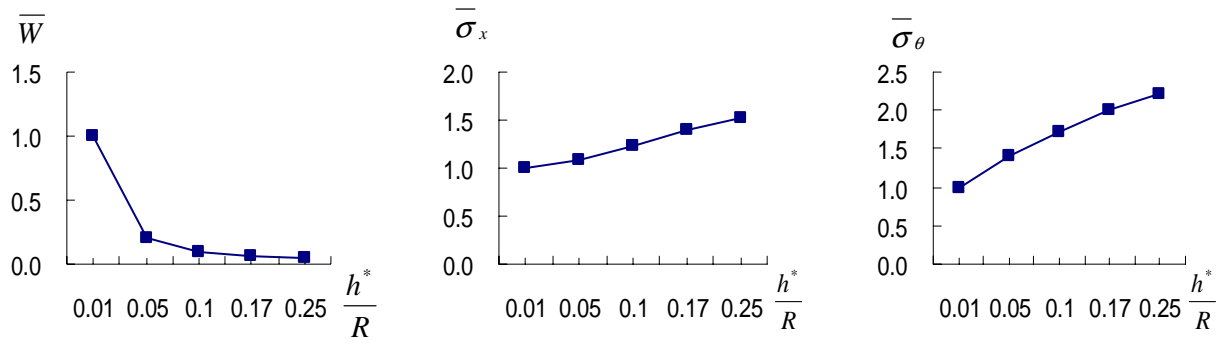
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Figure 4 $\dfrac{h^*}{R}$ v.s $\overline{W}, \overline{\sigma}_x, \overline{\sigma}_\theta$ at center position of hybrid shell Type1 under loads case2 and $\dfrac{a}{R} = 4$
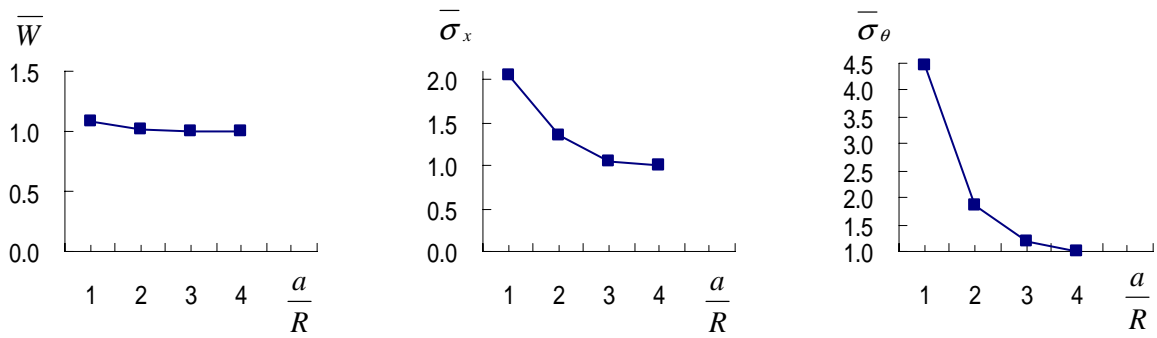
Figure 5 $\dfrac{a}{R}$ v.s $\overline{W}, \overline{\sigma}_x, \overline{\sigma}_\theta$ at center position of hybrid shell Type1 under loads case2 and $\dfrac{h^*}{R} = 0.01$
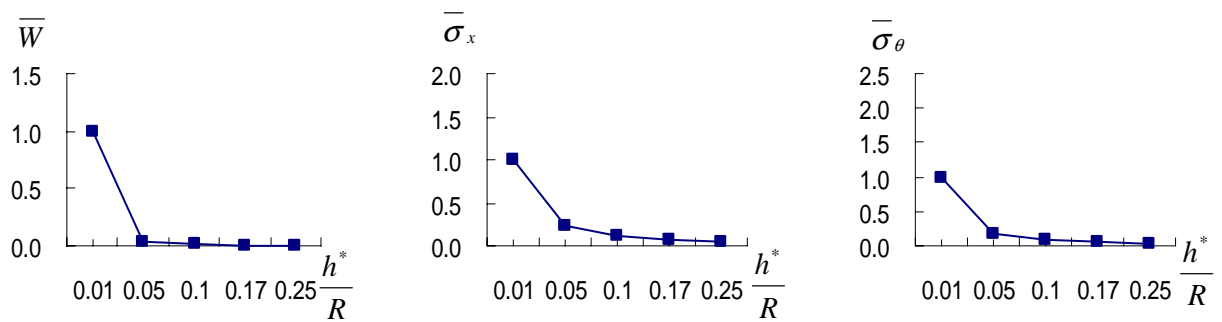
Figure 6 $\dfrac{h^*}{R}$ v.s $\overline{W}, \overline{\sigma}_x, \overline{\sigma}_\theta$ at center of $x-\theta$ plane and $\dfrac{Z}{h^*} = -0.1$ of hybrid shell Type2 under loads case1 and $\dfrac{h^*}{R} = 0.01$
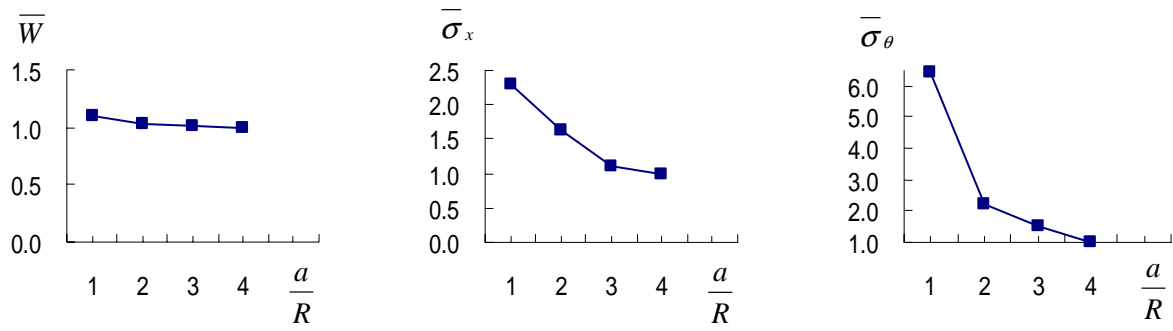
Figure 7 $\dfrac{a}{R}$ v.s $\overline{W},\overline{\sigma}_x,\overline{\sigma}_\theta$ at center of $x-\theta$ plane and $\dfrac{Z}{h^*}=-0.1$ of hybrid shell Type2 under loads case1 and $\dfrac{h^*}{R}=0.01$
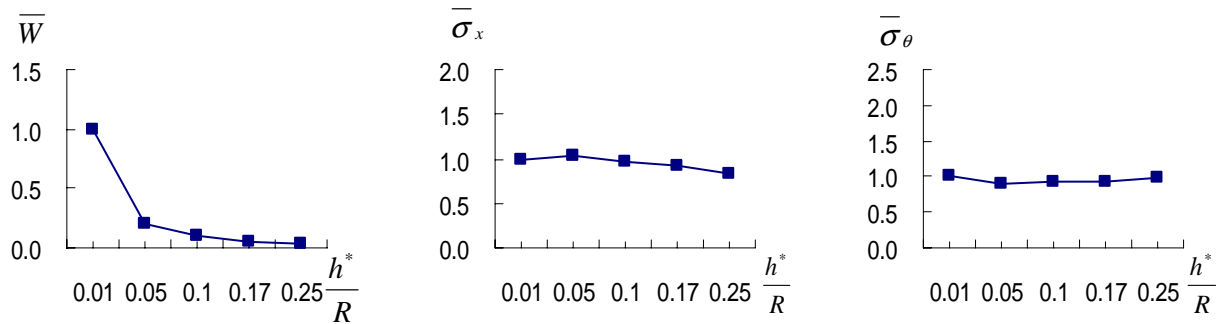

Figure 8 $\dfrac{h^*}{R}$ on $\overline{W},\overline{\sigma}_x,\overline{\sigma}_\theta$ at center of $x-\theta$ plane and $\dfrac{Z}{h^*}=-0.1$ of hybrid shell Type2 under loads case2 and $\dfrac{a}{R}=4$
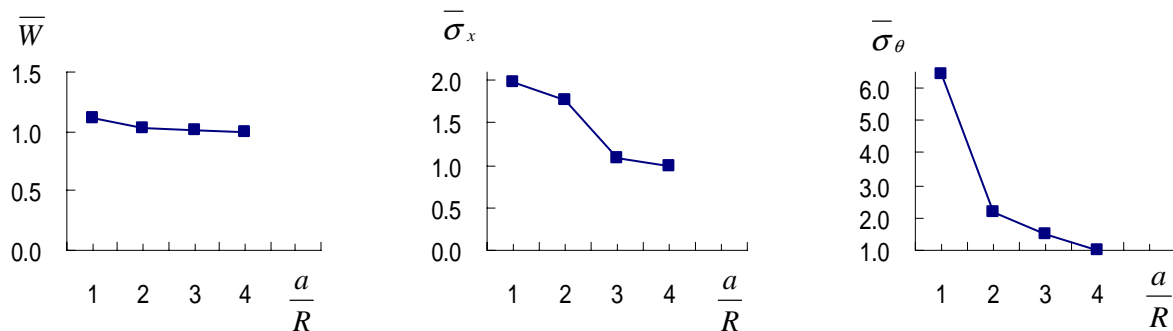

Figure 9 $\dfrac{a}{R}$ v.s $\overline{W},\overline{\sigma}_x,\overline{\sigma}_\theta$ at center of $x-\theta$ plane and $\dfrac{Z}{h^*}=-0.1$ of hybrid shell Type2 under loads case2 and $\dfrac{h^*}{R}=0.01$

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