Controlling Chaos in a Symmetric Gyroscope

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Abstract

In this paper, controlling chaos in a gyroscope has been studied. It has shown that one can convert chaotic motion to a regular one. For this purpose, the feedback control, the addition of constant motor torque, the addition periodic force, and adaptive control algorithm (ACA) are used to control chaos. As a result, the chaotic system can be controlled effectively.

Key Words: gyroscope、controlling chaos、feedback control、adaptive control、Liapunov exponents。

對稱陀螺儀的渾沌控制

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摘 要

本文研究了陀螺儀發生渾沌運動時的控制問題，通過週期激振力、自適應控制和連續反饋控制來抑制、控制其渾沌運動。本文以最有效的李雅普諾夫指數來確認系統是否從渾沌運動狀態轉變到規則運動狀態。數值分析表明在適當的控制訊號下，陀螺儀的渾沌運動得到了很好的控制。

關鍵字：陀螺儀、渾沌控制、反饋控制、自適應控制、李雅普諾夫指數

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1. Introduction

Lorenz studied the strange changes in the atmosphere at which is the first example to research chaos in 1963[1]. During the past one and half decades, a large number of studies have shown that chaotic phenomena are observed in many physical systems, possessing nonlinearity [2,3]. It was also reported that the chaotic motion occurred in many nonlinear control system [4,5]. Several interesting dynamic behaviors of the gyro have been studied recently [6-8]. It has shown that the system exhibited both regular and chaotic motions.

The presence of chaotic behavior is generic for suitable nonlinearities, ranges of parameters and external forces, where one wishes to avoid or control so as to improve the performance of a dynamical system. Sometimes chaos is useful, as in mixing process or in heat transfer, often it is unwanted or undesirable. Because of chaotic motion is unpredictable in time and the fracture character of the system depends on its sensitivity to initial conditions. Knowledge is valid in starting for near initial conditions in phase space as distance between the two points corresponding to the initial conditions. Due to unpredictability this knowledge is lost in time. Clearly, the ability to control chaos, that is to convert chaotic oscillations into desired regular ones with periodic time dependence, would be beneficial in working with a particular system. It is thus of great practical importance to develop suitable control methods. Very recently much interest has been focused on this type of problem — controlling chaos [9-12]. The aim of this paper is to control chaotic motion of the gyroscope. For this purpose, many control methods; which are the addition of periodic force, simple feedback control, the addition of constant motor force, and adaptive control; are used to control chaos. As a result, the chaotic system can be controlled.

2. Liapunov Exponent

The Liapunov exponent plays a major role when the control methods were determined, in this paper. The Liapunov exponent is the powerful index which clould be distinguished regular motion from chaotic motion. The Liapunov exponent may be used to measure the sensitive dependence upon initial conditions. It is an index for chaotic behavior. Different solutions of dynamical system, such as fixed points, periodic motions, quasi-periodic motion, and chaotic motion can be distinguished from it. If two trajectories start close to one another in phase space, they will move exponentially away from each other for small times on the average [13]. Thus, if d_0 is a measure of initial distance between the two starting points, the distance is $d(t)= d_0\, 2^{\lambda t}$. The symbol λ is called Liapunov exponent. The divergence of chaotic orbits can only be locally exponential, because if the system is bounded, as most physical experiments are, $d(t)$ can't grow to infinity. A measure of divergence of orbit is that the exponential grown at many points along a trajectory has to be averaged. When $d(t)$ is too large, a new 'nearby' trajectory $d_0(t)$ is defined. The Liapunov exponent can be expressed as:

$$\lambda = \frac{1}{t_N - t_0} \sum_{k=1}^{N} \log_2 \frac{d(t_k)}{d_0(t_k - 1)} \tag{1}$$

The signs of the Liapunov exponents provide a qualitative picture of a system dynamics. The criterion is

$$\lambda > 0 \text{ (chaotic)} \tag{2}$$

$$\lambda \le 0 \text{ (regular motion)} \tag{3}$$

3. Controlling Chaos by Several Methods

In this section, several methods are applied to study the controlling chaos. The regular and chaotic motions could be distinguished from the Liapunov exponents. If one of the valves of Liapunov exponents is greater than zero, it is chaotic motion, otherwise regular motion.

3.1 Suppression of Chaos by Simple Feedback Control

State feedback control is one of the simple methods for the control system, this control method is often used to design the controller. In this subsection, it is used to control the chaotic motion. The governing equation of motion of a symmetric dissipative gyro mounted on a vibrating base can be written as equation (4). Where I_1 and I_3 are the polar and equatorial moments of inertia of the symmetric gyro, respectively, Mg is the gravity force, $\bar{\ell}$ is the amplitude of the external excitation disturbance, and ω is the frequency of the external excitation disturbance, m is the mass of the damper, and k is the spring constant, p is a positive constant. The detailed explanation is referred to ref.[8]. The feedback control law is assumed to be

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{\beta_\phi^2}{[I_1 + m(x_3+p)^2]^2}\dfrac{(1-\cos x_1)^2}{\sin^3 x_1} + \dfrac{[(Mg\ell + mgp) + mgx_3 + (M+m)g\bar{\ell}\sin\omega t]\sin x_1}{[I_1 + m(x_3+p)^2]} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = (\dfrac{\beta_\phi^2}{[I_1 + m(x_3+p)^2]^2}\dfrac{(1-\cos x_1)^2}{\sin^2 x_1})(x_3+p) + g(1-\cos x_1) - \dfrac{k}{m}x_3 + (x_3+p)x_2^2 - 2cx_4 \end{cases} \quad (4)$$

$$U = -Kx_2, \quad (5)$$

which is added into eq. (4). It shows that the maximal Liapunov exponents can be less than 0 when the value of K is large sufficiently. Then the chaotic motion disappears, and the regularity returns. The maximal Liapunov exponents versus the values of K are shown in Fig 1(a).

3.2 Controlling of Chaos by the Addition of Constant Motor Torque

Interestingly, one can even add just a constant term to control or quench the chaotic attractor to a desired periodic one in typical nonlinear non-autonomous system. It ensures effective controlling in a very simple way. In order to understand this simple controlling approach in a better way, the system (4) will be integrated numerically.

In the absence of the constant motor torque, the system exhibits chaotic behavior under the following parameter conditions:I_1=1.0, k=100, ℓ=0.1 $\bar{\ell}$=5.0, M=0.5, m=0.1, p=0.1, β_ϕ^2 =100, ω=2.0.

Examining the effect of the constant motor torque, the added motor torque M_t is assumed to be in eq. (4). Now if we consider the effect of the constant motor torque M_t by increasing it from zero upwards, the chaotic behavior is then altered. Spectral analysis of the Liapunov exponents has proven to be the most useful dynamical diagnostic tool for examining chaotic motions. Fig. 1(b) shows the maximal Liapunov exponents. It is clear that the system returns to regular motion, when the constant motor torque M_t is presented in certain interval.

3.3 Controlling of Chaos by the Addition of Periodic Force

One can control system dynamics by addition of an external periodic force in the chaotic state. For our propose, the added periodic force, asin(ϖt), is assumed to be presented in eq. (4b). When the added periodic force is given, the system (4) then can be investigated by numerically solving, with the remaining parameters fixed. To examine the change in the dynamics of the system as a function of ϖ for fixed a=2, the maximal Liapunov exponents are estimated numerically. An interesting phenomenon is found that when $1.1 \leq \varpi \leq 1.3$, the maximal Liapunov exponents $\lambda_i \leq 0$.

3.4. Controlling chaos by the Adaptive Control Algorithm

Recently, Huberman and Lumer have suggested a simple and effective adaptive control algorithm (for one-dimensional systems), which utilizes an error signal proportional to the difference between the goal output and actual output of the system. The error signal governs the change of parameters of the system, which readjust so as to reduce the error to zero. The schematic diagram of an adaptive control system is depicted in Figure 2. This method can be explained briefly: The system motion is set back to a desired state X_S by adding dynamics on the control parameter P through the evolution equation,

$$\dot{P} = \varepsilon\, G(X - X_s), \tag{6}$$

where the function G is proportional to the difference between X_S and the actual output X, and ε indicates the stiffness of the control. The function G could be either linear or nonlinear. In order to convert the dynamics of a symmetric heavy gyroscope [6] from chaotic motion to a desired fixed point (X_{1S}, X_{2S}). The governing equation of motion of a symmetric heavy gyroscope is written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\alpha^2 \dfrac{(1-\cos x_1)^2}{\sin^3 x_1} - cx_2 \\ +\beta \sin x_1 + f \sin \omega t \sin x_1 \end{cases} \tag{7}$$

the detailed explanation is referred to ref.[6]. The chosen parameter f is perturbed as

$$\dot{f} = \varepsilon(X - X_s), \tag{8}$$

where the control function G is assumed to be linear in (X_1-X_{1S}) to start with. In this study, the fixed point is (0, 0), the stiffness of the control ε=0.1, X_{1S}=0 is the output of goal. When X approaches to X_S, then G((X_1-X_{1S}) tends to zero, i.e., G(0)=0. When t ≥ 1000, the control is switched on, as t ≥ 2050 the parameter f tends to 26.9 shown as Fig.3 (b), and the system evolves to reach the fixed point (0,0). Fig. 3(a) shows the difference between X_1 with X_{1S} (0). It is clear that the desired fixed point (0, 0) can reach by adaptive control algorithm.

4. Conclusions

Several control methods have been used to control the chaotic motion of the gyro. Some interesting phenomena have been found. An addition of sinusoidal force can eliminate chaos in a dynamical system. Regular motion is also recovered from chaotic state for appropriate feedback control law. A simple and high effective method, the addition of constant motor torque, has been used to suppress the chaotic motion. Besides it has shown that ACA can convert chaotic oscillations into desired regular goal.

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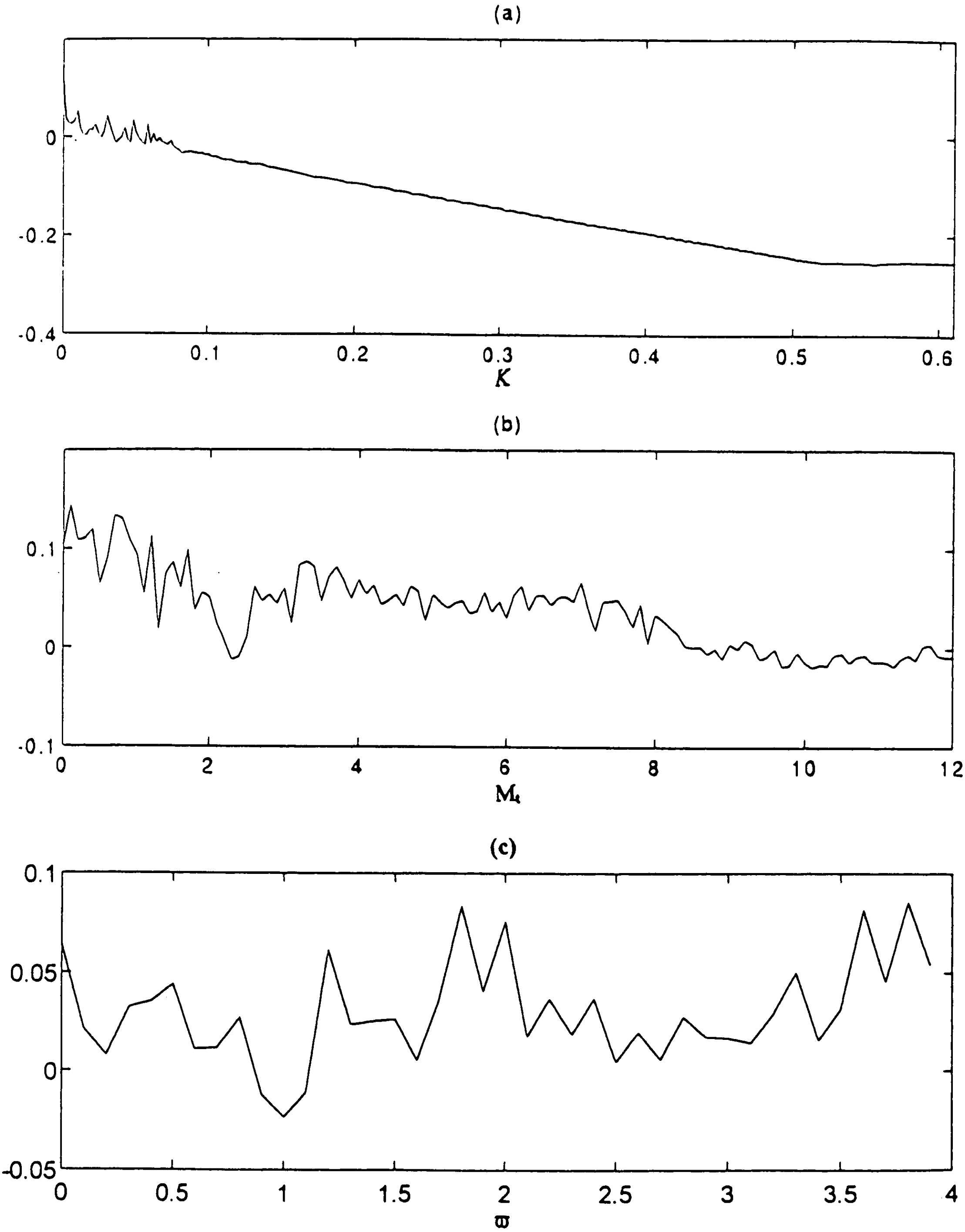

Fig.1 (a)Maximal Liapunov exponent against the parameter K; (b) Maximal Liapunov exponent against the parameter M_t; (c) Maximal Liapunov exponent against the parameter ϖ.

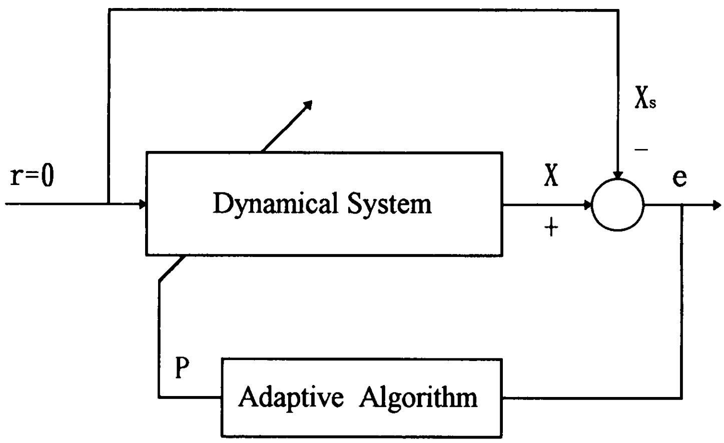

Fig.2 Schematic diagram of an adaptive control system. X_s is the desired output, X is the actual output, and e is the error signal.

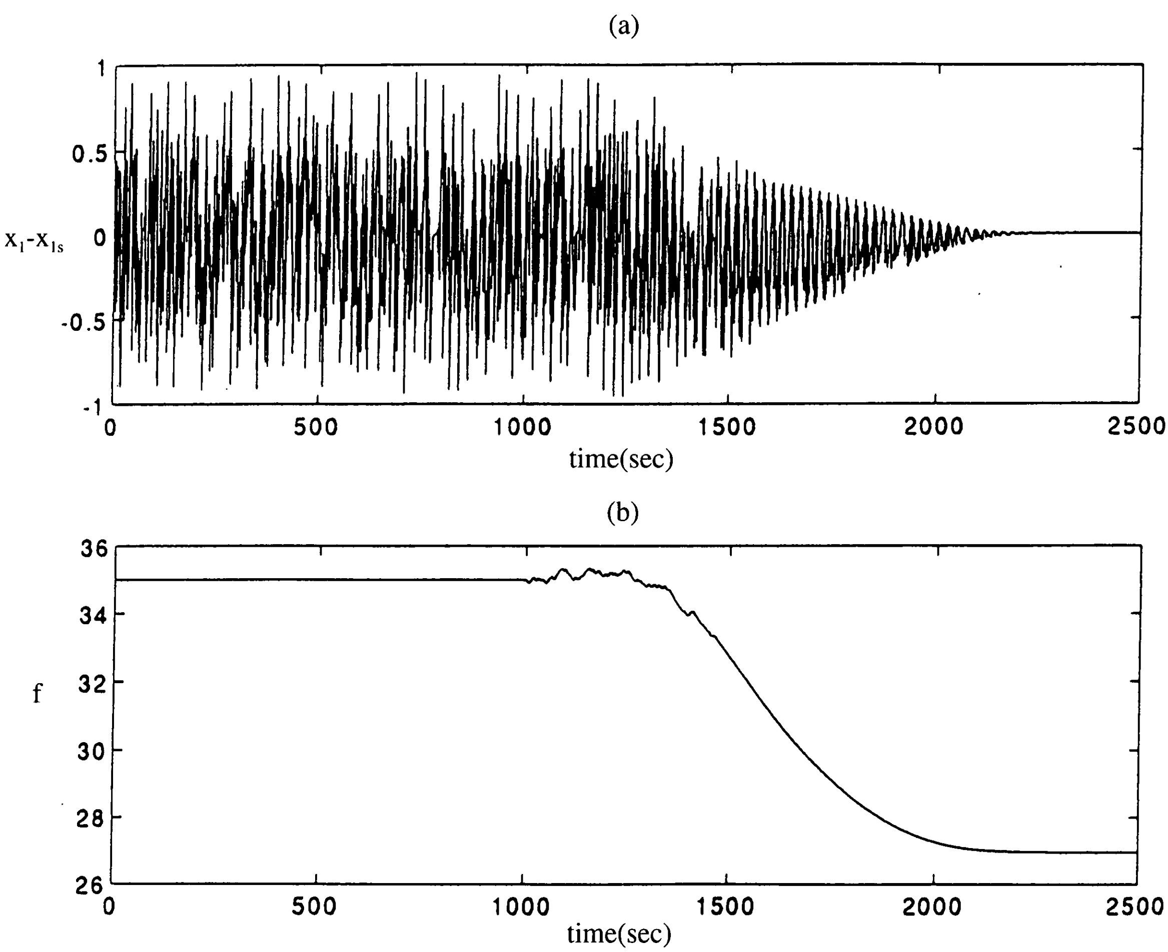

Fig.3 (a)The difference between the actual output x_1 with the desired output x_{1s}, (b)Variation of the control parameter f.