電動車推進系統的模糊建模及控制中使用線性矩陣不等式

謝承達

摘 要

本文提出一能在不同路況下達到速度追蹤的電動車推進系統模糊控制設計法。首先以T-S模糊模式近似使用高效率交流馬達的電動車推進系統的狀態方程式，然後根據T-S模糊模式進行強健模糊控制設計，使電動車推進系統具強健穩定性能。將控制設計問題轉換成線性矩陣不等式(LMI)問題凸出最佳化技術有效率地解出。由電腦模擬可驗證所提控制策略的有效性。

關鍵詞：電動車推進系統、T-S模糊模式、強健模糊控制、強健穩定化、凸出最佳化技術。

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T-S Fuzzy Modeling and Control for Electric Vehicle Propulsion Using Linear Matrix Inequality

Cheng-Da Hsieh

Abstract

This paper presents a fuzzy control design approach which can meet the speed tracking requirement when electric vehicles are operated on various traffic conditions. A T-S fuzzy model for approximating the state equation of an electric vehicle propulsion system with high energy efficiency-based ac motor drive is first proposed, and a robust fuzzy control based on the T-S fuzzy model is then considered. The robust stabilization for the EV(electric vehicle) propulsion system is cast into a linear matrix inequality (LMI) problem via roust performance analysis, and the LMI problem can be solved efficiently by using the convex optimization techniques. Computer simulations are presented for illustrating the performance of the suggested control strategy.

Key words: EV propulsion, T-S fuzzy model, robust fuzzy control, robust stabilization, linear matrix inequality, convex optimization techniques.

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1. Introduction

One dominant issue of EV design is to lengthen the running distance on one battery charge. This implies the importance of a high-efficiency motor drive. As enhanced insulated gate bipolar transistors (IGBT's) are used in the PWM inverter, the loss of the inverter is negligible. Mutoh et al. [1] proposes an energy saving strategy for induction motors. The rotor and stator copper losses and the core loss are minimized, and the optimal ratio of magnetizing current to the torque current is derived.

The stability and robustness problem of the EV propulsion control systems is also an important topic. Variable structure control using a proper switching law can drive the system into the predetermined sliding mode, and according to the sliding mode, the system can approach to its equilibrium. Thus, the sliding mode control approach can offer many good properties, such as insensitivity to parameters variation, external disturbance rejection, and fast dynamic response [2]. One more interesting design method of stabilization strategy for complex nonlinear systems can be as follows: first build a T-S fuzzy model for approximating the nonlinear plant, and a fuzzy model-based controller is then synthesized utilizing the concept of "parallel distributed compensation". Local linear feedback controls can be designed systematically by a generalized Lyapunov function and some linear matrix inequalities, and the closed-loop fuzzy control system composed of the fuzzy model and the PDC controller is globally asymptotically stable [3,11].

In this paper, based on the optimal relationship of the magnetizing current and the torque current found by the energy-saving control principle [1], the dynamics model and a T-S fuzzy model for EV propulsion systems with 3-phase AC induction motor (IM) are first constructed. Then, the parallel distributed compensation (PDC) approach [3] is adopted for synthesizing a robust speed tracking control for EV propulsion systems. Through the robust performance analysis for disturbance rejection the control problem is translated into a linear matrix inequality problem. By considering it as a generalized eigenvalue minimization problem (GEMP), the required common Lyapunov matrix and the local feedback gain matrices can be decided. The derived LMI-based control law can guarantee the stability and robustness of an EV propulsion system when it is driven from standstill to stable cruise under various uncertainties.

The paper is organized as follows: Section 2 presents the dynamics model of an energy-saving EV propulsion system. In Section 3, a T-S fuzzy model for the EV propulsion system is proposed, and a LMI-based robust speed control design using the derived T-S fuzzy model is suggested. Some representative simulation results are shown in Section 4. Finally, conclusions are made in Section 5.

2. Modeling of an Electric Vehicle Propulsion System

Consider the power train for an EV shown in Fig.1, where an ac induction motor is used for generating the driving torque. The dynamics model for the load part consisting of reduction gears and a differential gear for driving the rear wheels of an EV will be derived by the first principles. The ac induction motor is assumed with the energy saving driving strategy proposed by [1], and the complete mathematical model for the power train will be constructed.

The angular displacements of the motor, the rear shaft and the rotor of the rear wheels are defined as θ_m, θ_1 and θ_2, respectively, as shown in Fig. 1. Let the transmission ratio of the reduction gear

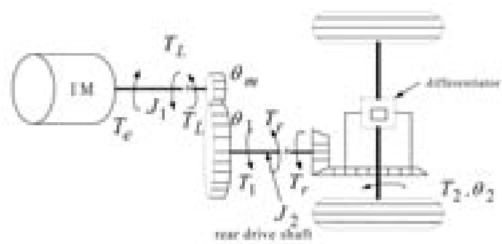


Fig. 1 Schematic of the power train for EV.(IM + reduction gear + differential gear)

be $N_0=\theta_m/\theta_1$, and the reduction ratio for the differential gears be $N_d=\theta_1/\theta_2$. Then the load torque (T_l) equation can be derived below.

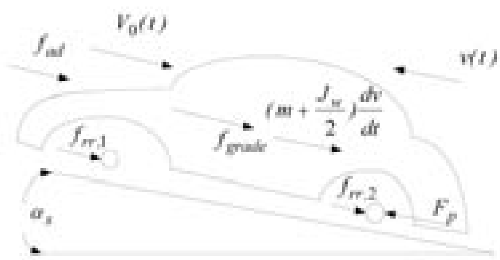


Fig. 2 Force diagram in the longitudinal direction of an EV.

Refer to Fig. 2, the equivalent propulsion force F_P generated by the EV drive system and the equation of motion of the vehicle can be expressed as :

$$f_{ar} + f_{ad} + f_{grade} + f_{rr} = F_p = T_2 / r \tag{1}$$

where T_2 is the driving torque on the rear wheel shaft (as shown in Fig. 1), and r is the wheel radius. The inertia resistance f_{ar} can be derived as:

$$f_{ar} = m\frac{dv}{dt} + \frac{J_w}{r}\frac{d\omega}{dt} = m\frac{dv}{dt} + \frac{J_w}{r}\frac{d(v/r)}{dt} = \left(m + \frac{J_w}{r^2}\right)\frac{dv}{dt} \tag{2}$$

where v is the velocity of the vehicle, $J_w = J_{w1} + J_{w2}$ (J_{w1} and J_{w2} are respectively the moments of inertia of the front and rear wheel shaft systems) is the total moment of inertia of the rotation parts, and $m = m_B + m_1 + m_2$ is the total mass, here m_B , m_1 , and m_2 are respectively the mass of the vehicle body, the front and rear wheel shaft systems.

The aerodynamic drag is given as:

$$f_{ad} = 0.5\zeta C_w A(v+V_0)^2 \operatorname{sgn}(v+V_0) \tag{3}$$

where ζ is the air density, C_w is the aerodynamic drag coefficient, A is the vehicle frontal area, V_o is the head-wind velocity, $v+V_o$ is the velocity of the vehicle relative to the head-wind, and sgn is the sign function. The grade resistance is

$$f_{grade} = mg\sin\alpha_s \tag{4}$$

where α_s is the grade angle, α_s is positive for the upgrade case and negative for the downgrade case. The rolling resistance f_{rr} composed of the front and rear parts ($f_{rr,1}$ and$f_{rr,2}$) can be expressed as:

$$f_{rr} = Kmg\,\mathrm{sgn}\,v \tag{5}$$

where $K = 0.014\left(1 + v^2/1500\right)$ is the tire rolling resistance coefficient [6,7], and is the gravity constant. By Eq. (1), we can obtain

$$T_2 = r\begin{bmatrix} \left(m + \dfrac{J_w}{r^2}\right)\dfrac{dv}{dt} + 0.5\zeta C_w A(v+V_0)^2\,\mathrm{sgn}(v+V_0) \\ + mg(\sin\alpha_s + K\,\mathrm{sgn}v) \end{bmatrix} \tag{6}$$

Since (refer to Fig. 1)

$$\frac{Tr}{T_2} = \frac{\theta_2}{\theta_1} = \frac{1}{N_d}, \quad \frac{T_l}{T_1} = \frac{\theta_1}{\theta_m} = \frac{1}{N_0} \tag{7}$$

we have

$$\begin{aligned} T_l &= \frac{T_1}{N_0} = \frac{1}{N_0}\left(J_2\ddot{\theta}_1 + T_r\right) = \frac{1}{N_0}\left(J_2\ddot{\theta}_1 + \frac{T_2}{N_d}\right) \\ &= \left[\frac{J_2}{N_0^2} + \frac{mr^2 + J_w}{(N_0N_d)^2}\right]\ddot{\theta}_m(t) + \frac{0.5\zeta C_w A r^3}{(N_0N_d)^3}\dot{\theta}_m^2(t)\,\mathrm{sgn}(v+V_0) \\ &+ \frac{\zeta C_w A r^2}{(N_0N_d)^2}V_0(t)\dot{\theta}_m(t)\,\mathrm{sgn}(v+V_0) \\ &+ \frac{0.5\zeta C_w A r}{N_0N_d}V_0^2(t)\,\mathrm{sgn}(v+V_0) + \frac{rmg}{N_0N_d}(K\,\mathrm{sgn}v + \sin\alpha_s) \end{aligned} \tag{8}$$

$$v_{qs}^{rf} = r_s i_{qs}^{rf} + p\lambda_{qs}^{rf} + \omega_{rf}\lambda_{ds}^{rf} \tag{9}$$

$$v_{ds}^{rf} = r_s i_{ds}^{rf} + p\lambda_{ds}^{rf} - \omega_{rf}\lambda_{qs}^{rf} \tag{10}$$

$$0 = r_r i_{qr}^{rf} + p\lambda_{qr}^{rf} + (\omega_{rf} - \omega_r)\lambda_{dr}^{rf} \tag{11}$$

$$0 = r_r i_{dr}^{rf} + p\lambda_{dr}^{rf} - (\omega_{rf} - \omega_r)\lambda_{qr}^{rf} \tag{12}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda_{dr}^{rf} i_{qs}^{rf} - \lambda_{qr}^{rf} i_{ds}^{rf}) \tag{13}$$

where

$$\lambda_{ds}^{rf} = L_{ls} i_{ds}^{rf} + L_m(i_{ds}^{rf} + i_{dr}^{rf}) \tag{14}$$

$$\lambda_{qs}^{rf} = L_{ls} i_{qs}^{rf} + L_m(i_{qs}^{rf} + i_{qr}^{rf}) \tag{15}$$

$$\lambda_{dr}^{rf} = L_{lr} i_{dr}^{rf} + L_m(i_{ds}^{rf} + i_{dr}^{rf}) \tag{16}$$

$$\lambda_{qr}^{rf} = L_{lr} i_{qr}^{rf} + L_m(i_{qs}^{rf} + i_{qr}^{rf}) \tag{17}$$

here $p=d/dt$; v_{qs}^{rf} and v_{ds}^{rf} are the stator's input voltage components along the *q*- and *d*- axes of the rotor flux frame, respectively; λ_{qs}^{rf} and λ_{ds}^{rf} are the stator's flux linkage components along the *q*- and *d*- axes of the *rf* frame, respectively; r_s and r_r are the resistances of the stator and rotor, respectively; ω_{rf} is the rotating velocity of the *rf* frame; ω_r is the rotating velocity of the rotor; i_{qr}^{rf} and i_{dr}^{rf} are the rotor's current components along the *q*- and *d*- axes of the

rf frame, respectively; λ_{qr}^{rf} and λ_{dr}^{rf} are the rotor's flux linkage components along the q- and d- axes of the rf frame, respectively; T_e is the electromagnetic torque of the motor; P is the number of poles; L_m is the magnetization inductance; L_r is the rotor's inductance; and L_{ls} and L_{lr} are the leakage inductances of the stator and rotor, respectively.

By the principle of field orientation and letting the d- axis be entirely aligned with the rotor flux, we have, $\lambda_{qr}^{rf} = 0$, and the torque equation can be simplified as

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\lambda_{dr}^{rf} i_{qs}^{rf} \tag{18}$$

In order to generate the motor torque with maximum efficiency, the total loss P_l in the drive system must be a minimum. The total loss P_l generated in the drive system of an electric vehicle can be summarized as follows [1]:

$$P_l = P_{l1} + P_{INV} + P_{MEC} + P_{STR} \tag{19}$$

where

$$P_{l1} = \frac{3}{2}\left\{ \begin{array}{l} r_s\left(\left(i_{ds}^{rf}\right)^2 + \left(i_{qs}^{rf}\right)^2\right) + r_r\left(\left(i_{dr}^{rf}\right)^2 + \left(i_{qr}^{rf}\right)^2\right) + \\ r_m\left(\left(i_{ds}^{rf}\right)^2 + \left(i_{dr}^{rf}\right)^2\right) \end{array} \right\} \tag{20}$$

here r_m is the core loss resistance, P_{STR} is the stray load loss, P_{MEC} is the mechanical loss, and P_{INV} is the inverter loss. Only the losses P_{l1} and P_{INV} can be controlled in the ac drive design. The loss P_{INV} is very small in comparison with the loss P_{l1}, as long as enhanced insulated gate bipolar transistors (IGBTs) are used in the PWM inverter. In this case, the efficiency of the inverter is generally more than 95%. Thus, the P_{INV} loss can be neglected.

Since $\lambda_{qr}^{rf} = 0$, by Eqs. (12) and (17), we have

$$\begin{aligned} i_{dr}^{rf} &= -\frac{1}{r_r}\frac{d\lambda_{dr}^{rf}}{dt} \\ i_{qr}^{rf} &= -\frac{L_m}{L_r} i_{qs}^{rf} \end{aligned} \tag{21}$$

Usually, the response of λ_{dr}^{rf} is much slower than those of i_{qs}^{rf} and i_{ds}^{rf} , thus i_{dr}^{rf} almost equals zero. Hence P_l can be simplified as

$$P_{l1} = \frac{3}{2}\left[(r_s + r_m)(i_{ds}^{rf})^2 + (r_s + r_r')(i_{qs}^{rf})^2\right] \tag{22}$$

$$r_r' = r_r\left(\frac{L_m}{L_r}\right)^2 \tag{23}$$

Substituting (16) into (18), we have

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(L_m i_{ds}^{rf})(i_{qs}^{rf}) = K_T i_{ds}^{rf} i_{qs}^{rf} \tag{24}$$

where $K_T = \frac{3}{2}\frac{P}{2}\frac{L_m^2}{L_r}$ is the motor torque constant.

Let α be the ratio of the magnetizing current to the torque current, i.e.,

$$\alpha = \frac{i_{ds}^{rf}}{i_{qs}^{rf}} \tag{25}$$

Then P_{l1} can be expressed in terms of α :

$$P_{l1} = \frac{3}{2}\frac{T_e}{K_T}\left(A\alpha + \frac{B}{\alpha}\right) \tag{26}$$

$$A = r_s + r_m, B = r_s + r_r'$$

The optimal ratio α_{min} that makes the loss P_1 a minimum can be derived by letting $dP_1/d\alpha = 0$:

$$\alpha_{\min} = \sqrt{\frac{r_s + r_r'}{r_s + r_m}} \tag{27}$$

Substituting

$$i_{ds}^{rf} = \alpha_{\min} i_{qs}^{rf} \tag{28}$$

into the motor torque equation, we have

$$T_e = K_T \alpha_{\min} (i_{qs}^{rf})^2 \tag{29}$$

The dynamics model for the EV propulsion system shown in Fig. 1 can thus be derived as:

$$J_1 \frac{d\omega_m}{dt} + T_l = T_e \tag{30}$$

where ω_m is the angular velocity of the motor rotor in rad/s, J_l is the moment of inertia of the motor shaft including the reduction gear in this side. By substituting (8) into (30), we have the complete dynamics model as follows:

$$
\begin{aligned}
&\left[J_1 + \frac{J_2}{N_0^2} + \frac{mr^2 + J_w}{(N_0 N_d)^2}\right]\ddot{\theta}_m(t) \\
&+ \frac{0.5\zeta C_w A r^3}{(N_0 N_d)^3}\dot{\theta}_m^2(t)\operatorname{sgn}(v + V_0) \\
&+ \frac{\zeta C_w A r^2}{(N_0 N_d)^2} V_0(t)\dot{\theta}_m(t)\operatorname{sgn}(v + V_0) \\
&+ \frac{0.5\zeta C_w A r}{N_0 N_d} V_0^2(t)\operatorname{sgn}(v + V_0) \\
&+ \frac{rmg}{N_0 N_d}\left(K \operatorname{sgn} v + \sin\alpha_s\right) = T_e(t)
\end{aligned}
\tag{31}
$$

3. Fuzzy Model-Based Control Design of Electric Vehicle Propulsion System

The LMI techniques will be applied to the stability analysis of an electric vehicle propulsion control system. Defining the state variables as: $x_1 = \theta_m, x_2 = \dot{\theta}_m$, Eq. (31) can be rewritten as:

$$
\begin{aligned}
\dot{x}_1 &= \dot{\theta}_m = x_2 \\
\dot{x}_2 &= \ddot{\theta}_m = -\frac{C_1^3 C_2}{A_1} x_2^2 \operatorname{sgn}(v + V_0) \\
&\quad - \frac{2C_1^2 C_2}{A_1} V_0(t) x_2 \operatorname{sgn}(v + V_0) \\
&\quad - \frac{C_1 C_2}{A_1} V_0^2(t)\operatorname{sgn}(v + V_0) \\
&\quad - \frac{C_1}{A_1} mg\left(K \operatorname{sgn} v + \sin\alpha_s\right) + \frac{1}{A_1} T_e
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
T_e &= K_T i_{ds}^{rf} i_{qs}^{rf} = \alpha_{\min} K_T \left(i_{qs}^{rf}\right)^2 \\
A_1 &= J_1 + \frac{J_2}{N_0^2} + \frac{mr^2 + J_\omega}{N_0 N_d} \\
C_1 &= \frac{r}{N_0 N_d},\ C_2 = 0.5\zeta C_\omega A
\end{aligned}
$$

α_S and V_O are considered as with uncertainty. After some manipulations, (32) can be rewritten as

$$
\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}u + \boldsymbol{D} + \boldsymbol{p}_u \tag{33}
$$

where $\boldsymbol{x} = [x_1\ x_2]^T = [x_1\ \dot{x}_1]^T$ is the state vector; $u = \left(i_{qs}^{rf}\right)^2$ is the input variable; $\boldsymbol{A}(x) = \begin{bmatrix} 0 & 1 \\ 0 & ax_2 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ \alpha_{\min} K_T / A_1 \end{bmatrix}$, $\boldsymbol{D} = \begin{bmatrix} 0 \\ d_s \end{bmatrix}$, with $a = \frac{1}{A_1}\left[-C_1^3 C_2 \operatorname{sgn}(v + V_0) - 0.9333 \times 10^{-5}\left(mgC_1^2\right)\operatorname{sgn} v\right]$ and and $d_s = \frac{1}{A_1}\left(-0.014 C_1 mg\right)\operatorname{sgn} v$, and $\boldsymbol{p}_u$ and p_u including all the other terms is considered as uncertainty. Without loss of generality, we consider the major operating case of sng(v+V_O)=1 and sng v=1

Since B is only a constant matrix, the derivation of a T-S fuzzy model and then the control design and finding of the solution of LMIs become much less difficult than the approaches directly based on the original complex model.

Assume $x_2 \in [\omega_{\min}, \omega_{\max}]$ for the nonlinear terms in Eq. (33). Defining $z=X_2$, the maximal and minimal values of z can be deduced and expressed as below:

$$\max_{x_2} z(t) = \omega_{max}, \quad \min_{x_2} z(t) = \omega_{\min}$$

Choose z as the antecedent variable of the T-S fuzzy model, we can define two fuzzy sets with membership functions shown in Fig. 3 in the universe of discourse of z. Then a T-S fuzzy model can be constructed analytically as follows:

Model Rule i:

$$\begin{aligned} &\text{IF} \quad z \quad \text{is} \quad C^i \\ &\text{THEN} \quad \dot{\mathbf{x}} = \mathbf{A}_i\mathbf{x} + \mathbf{B}u + \mathbf{D} + \mathbf{p}_u, \quad i = 1,2 \end{aligned} \tag{34}$$

where $C^1 = M_1, C^2 = M_2$;

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 \\ 0 & a\omega_{\max} \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ 0 & a\omega_{\min} \end{bmatrix}$$

The overall equation of the T-S fuzzy model can be inferred as

$$\dot{\mathbf{x}} = \sum_{i=1}^{2} h_i(z)\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}u + \mathbf{D} + \mathbf{p}_u \tag{35}$$

where

$$h_i(z) = \frac{w_i(z)}{\sum_{i=1}^{2} w_i(z)} \tag{36}$$

$$w_i(z) = C^i(z), \quad i = 1,2$$

here C^i (z) is the grade of membership of z in fuzzy set C^i. By the membership function definitions shown in Fig. 3, we have

$$\begin{aligned} &h_i(z) \geq 0, \ i = 1,2, \\ &\sum_{i=1}^{2} h_i(z) = 1 \end{aligned} \tag{37}$$

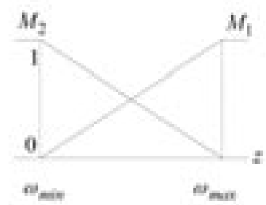


Fig. 3 Definition of two membership

For arbitrary trajectory tracking control, first define the tracking error vector as

$$\mathbf{e} = \mathbf{x} - \mathbf{x}_d \tag{38}$$

where $\mathbf{x}_d = [x_{d1} \quad x_{d2}]^T = [\theta_d \quad \omega_d]^T$ is the desired trajectory vector. Notice that $\theta_d(t)$ and $\omega_d(t)$ are the desired angular displacement and velocity trajectories of the rotor about its rotating axis, respectively. Differentiating Eq. (38), we have

$$\dot{\mathbf{x}} = \dot{\mathbf{e}} + \dot{\mathbf{x}}_d \qquad (39)$$

Substituting (39) into the model rules (34), we can obtain the T-S fuzzy model for the error dynamics as follows:

Error Rule i :

If z is C^i

Then

$$\dot{\mathbf{e}} = \mathbf{A}_i\mathbf{e} + \mathbf{B}u + \mathbf{D} + \mathbf{p}_u + \mathbf{A}_i\mathbf{x}_d - \dot{\mathbf{x}}_d, \qquad i = 1,2 \qquad (40)$$

The output of the error T-S fuzzy model can be defuzzified as:

$$\dot{\mathbf{e}} = \sum_{i=1}^{2} h_i(z)(\mathbf{A}_i\mathbf{e} + \mathbf{A}_i\mathbf{x}_d) + \mathbf{B}u + \mathbf{D} + \mathbf{p}_u - \dot{\mathbf{x}}_d \qquad (41)$$

In this study, the PDC(Parallel Distributed Compensation)[3,11]with control rules constructed based on the T-S fuzzy model rules is adopted for the fuzzy control design. Each control rule has a linear state feedback part and a feedforward part to compensate for the effect of gravity, that is,

Control Rule i :

If z is C^i

Then

$$u = -K_i\mathbf{e} - B^+\mathbf{A}_i\mathbf{x}_d - B^+(\mathbf{D} - \dot{\mathbf{x}}_d), \qquad i = 1,2 \qquad (42)$$

where $B^+ = [0\ \mathrm{B}_2^T(B_2B_2^T)^{-1}]$. So the design objective is to determine the local feedback gains K_i in the consequent parts of the control rules via LMI.

The output of the PDC controller can be inferred as:

$$u = -\sum_{i=1}^{2} h_i(Z)K_i\mathbf{e} - B^+\sum_{i=1}^{2} h_i(Z)\mathbf{A}_i\mathbf{x}_d - B^+(\mathbf{D} - \dot{\mathbf{x}}_d) \qquad (43)$$

By substituting (43) into (41), and since B is a constant matrix in the T-S fuzzy model and $\Sigma_{i=1}^{2} h_i(Z) = 1$, the error dynamics for the whole closed-loop system can be derived as

$$\dot{\mathbf{e}} = \sum_{i=1}^{2} h_i(z)(\mathbf{A}_i - \mathbf{B}K_i)\mathbf{e} + \mathbf{p}_u \qquad (44)$$

For the consideration of robustness with respect to the disturbance P_u , the following robust performance requirement [4] for the tracking error is to be met:

$$\frac{\int_0^{t_f} \mathbf{e}Q\mathbf{e}\,dt}{\int_0^{t_f} \mathbf{P}_u^T\mathbf{P}_u\,dt} < \rho^2 \tag{45}$$

where $Q = \alpha P, \alpha > 0$, and is a symmetric positive definite matrix. Equation (45) means that the effect of P_u on the error must be attenuated below a prescribed level ρ. To synthesize the fuzzy controller that can reject the external disturbances of an electric vehicle propulsion control system, we can select a positive definite function as follows:

$$V(\mathbf{e}) = \{\mathbf{e}^T\mathbf{P}\mathbf{e}\} \tag{46}$$

The requirement (45) for a prescribed ρ>0 can be shown to be equivalent to the following condition:

$$\dot{V} + \alpha e^T P e - \rho^2 P_u^T P_u \le 0 \tag{47}$$

By integrating (47) from 0 to t_f with initial condition e (0)=0,we have

$$V + \int_0^{t_f} (\alpha e^T P e - \rho^2 P_u^T P_u)dt \le 0 \tag{48}$$

Thus,

$$\int_0^{t_f} (\alpha e^T P e - \rho^2 P_u^T P_u)dt \le -V \le 0 \tag{49}$$

Equation (49) implies (45). Therefore if (47) holds, the robust performance requirement can be guaranteed under P_u .

The LMI constraints can be derived from (47). First, rewrite (47) as

$$\dot{\mathbf{e}}^T Pe + e^T P\dot{e} + \alpha e^T Pe - \rho^2 P_u^T P_u \le 0 \tag{50}$$

and substituting (44) into (50),we have

$$\sum_{i=1}^{2} h_i(z) e^T\left[(\mathbf{A}_i - \mathbf{B}K_i)^T P + P(\mathbf{A}_i - \mathbf{B}K_i)\right]e + \mathbf{p}_u^T Pe + e^T P\mathbf{p}_u + \alpha e^T Pe - \rho^2 \mathbf{p}_u^T \mathbf{p}_u \le 0 \tag{51}$$

That is,

$$\sum_{i=1}^{2} h_i(z)\begin{bmatrix} e \\ \mathbf{P}_u \end{bmatrix}^T \begin{bmatrix} (\mathbf{A}_i - \mathbf{B}K_i)^T P + P(\mathbf{A}_i - \mathbf{B}K_i) + \alpha P & P \\ P & -\rho^2 I \end{bmatrix} \begin{bmatrix} e \\ \mathbf{P}_u \end{bmatrix} \le 0 \tag{52}$$

Therefore, if the following constraints are satisfied:

$$\begin{bmatrix} -(\mathbf{A}_i - \mathbf{B}K_i)^T P - P(\mathbf{A}_i - \mathbf{B}K_i) - \alpha P & -P \\ -P & \rho^2 I \end{bmatrix} \ge 0, \quad i = 1,2 \tag{53}$$

then Equation(52) holds. Employing the Schur complements for nonstrict inequalities [5],(53) becomes

$$-(\mathbf{A}_i - \mathbf{B}K_i)^T P - P(\mathbf{A}_i - \mathbf{B}K_i) - \alpha P - \frac{1}{\rho^2} PP \geq 0,$$
$$i = 1,2 \tag{54}$$

Conditions (54) can be solved by considering it as a generalized eigenvalue minimization problem (GEMP), that is, to maximize α subject to the following constraints:

1. $P > 0$

2. $$(\mathbf{A}_i - \mathbf{B}K_i)^T P + P(\mathbf{A}_i - \mathbf{B}K_i) + \frac{1}{\rho^2} PP \leq -\alpha P,$$
$$i = 1,2 \tag{55}$$

Because the second inequalities in (55) are not jointly convex in P and K_i , it is difficult to find a common solution P and K_i . Fortunately, the inequalities can be transferred into matrix inequalities by variable transformation.

Defining new variable $X=P^{-1}$, and multiplying the inequalities on the left and right by X, we can obtain

$$X(\mathbf{A}_i - \mathbf{B}K_i)^T PX + XP(\mathbf{A}_i - \mathbf{B}K_i)X +$$
$$\frac{1}{\rho^2} XPPX \leq -\alpha XPX,\ i = 1,2 \tag{56}$$

Equation (56) can be rewritten as

$$X\mathbf{A}_i^T + \mathbf{A}_i X - (\mathbf{B}M_i)^T - \mathbf{B}M_i + \frac{1}{\rho^2} \leq -\alpha X,\ i = 1,2 \tag{57}$$

where $M_i \equiv K_i X$. That is, the PDC control design problem can be transformed to the problem of maximizing α subject to the following linear matrix inequality constraints:

1. $X > 0$

2. $$X\mathbf{A}_i^T + \mathbf{A}_i X - (\mathbf{B}M_i)^T - \mathbf{B}M_i + \frac{1}{\rho^2} \leq -\alpha X,\ i = 1,2 \tag{58}$$

If there exists a common X and M_i 's satisfying the above LMI constraints, then the common P and K_i can be obtained as

$$P = X^{-1} \qquad \text{and} \qquad K_i = M_i P,\quad i = 1,2 \tag{59}$$

There exist methods in the literature for solving the LMI problems, such as interior point algorithm [5]. The MATLAB software package has incorporated this algorithm into the solver of LMI control toolbox. In this study, the instruction gevp is used to solve the above GEVP problem. Based on the proper choice of suitable α, P and the feedback gains $K_i, i=1,2$, can thus be determined, and the design of the control law (43) is accomplished.

4. Simulation Results

In this section computer simulations are used to illustrate the performance of the proposed T-S fuzzy model-based control strategy for an electric vehicle which is operated on various traffic conditions. The nominal values of the parameters used in the simulations are chosen as:

(1) Induction motor (60Kw, 2430rpm, 250N-m) and transmission with the following parameters:

$R_s = 61.4\mathrm{m}\Omega$, $R_r = 3.17\mathrm{m}\Omega$, $R_m = 16.1\mathrm{m}\Omega$,

$L_r = 0.0505\mathrm{mH}$, $L_s = 0.02856\mathrm{mH}$, $L_m = 1\mathrm{mH}$,

$J_1 = 1\mathrm{Kg\cdot m^2}$, $J_2 = 1.2\mathrm{Kg\cdot m^2}$, $J_{w1} = J_{w2} =$

$1.419\mathrm{Kg\cdot m^2}$.

(2) Road and load with following parameters:

$A = 1.5\mathrm{m^2}$, $C_w = 0.35$, $\zeta = 1.2258\,\mathrm{N\cdot s^2/m^2}$,

$r = 0.445\mathrm{m}$, $m = 525\mathrm{Kg}$, and

$\mu = 2.1553\times 10^{-5}$.

The operating range for $x_2(z)$ is set as:

$\omega_{min} =$ -62.8319 rad/sec , $\omega_{max} = 376.9911$ rad/sec .

The desired speed trajectory command, is selected as [12]:

$$\omega_d(t) = \begin{cases} \left(10 - \dfrac{15t}{t_f} + \dfrac{6t^2}{t_f^2}\right)\left(\dfrac{t^3}{t_f^3}\right)\omega_f & 0 \le t \le 10 \\ \omega_f & 10 < t \le 14 \\ \dfrac{(24-t)^3}{t_f^3}\left(10 - \dfrac{15(24-t)}{t_f} + \dfrac{6(24-t)^2}{t_f^2}\right)\omega_f & 14 < t \le 24 \\ 0 & \end{cases}$$

where ω_f =209.44 rad/sec is the desired maximum angular speed in 10<t≤14sec, and t_f = 10 sec.

Two traffic conditions are considered. Traffic condition (I) is selected as:

$$V_0(t)=\begin{cases}3t/4 & 2\le t\le 8\\ 3(10-t) & 8<t\le 10\\ 5 & 10<t\le 16\\ 2 & 16<t\le 19\\ 4 & 19<t\le 21\end{cases}$$

$$\alpha_s(t)=\begin{cases}\pi t/360 & 2\le t\le 8\\ 4\pi/180 & 8<t\le 10\\ (14-t)\pi/180 & 10<t\le 16\\ -2\pi/180 & 16<t\le 19\\ (t-18)\pi/180 & 19<t\le 21\end{cases}$$

Traffic condition (II) is selected as:

$$V_0(t)=\begin{cases}t & 2\le t\le 8\\ 6(10-t) & 8<t\le 10\\ 10 & 10<t\le 16\\ 2 & 16<t\le 19\\ 8 & 19<t\le 21\end{cases}$$

$$\alpha_s(t)=\begin{cases}\pi t/360 & 2\le t\le 8\\ 12\pi/180 & 8<t\le 10\\ (14-t)\pi/180 & 10<t\le 16\\ -4\pi/180 & 16<t\le 19\\ (t-18)\pi/180 & 19<t\le 21\end{cases}$$

Using the usual LMI method, the feedback gain matrices $\boldsymbol{K}_i, i=1,2,$ and the common positive definite matrix P can be obtained as follows:

$\mathbf{K}_1 = 10^7\times[3.4372, 2.0517]$,

$\mathbf{K}_2 = 10^7\times[3.4399, 2.053]$,

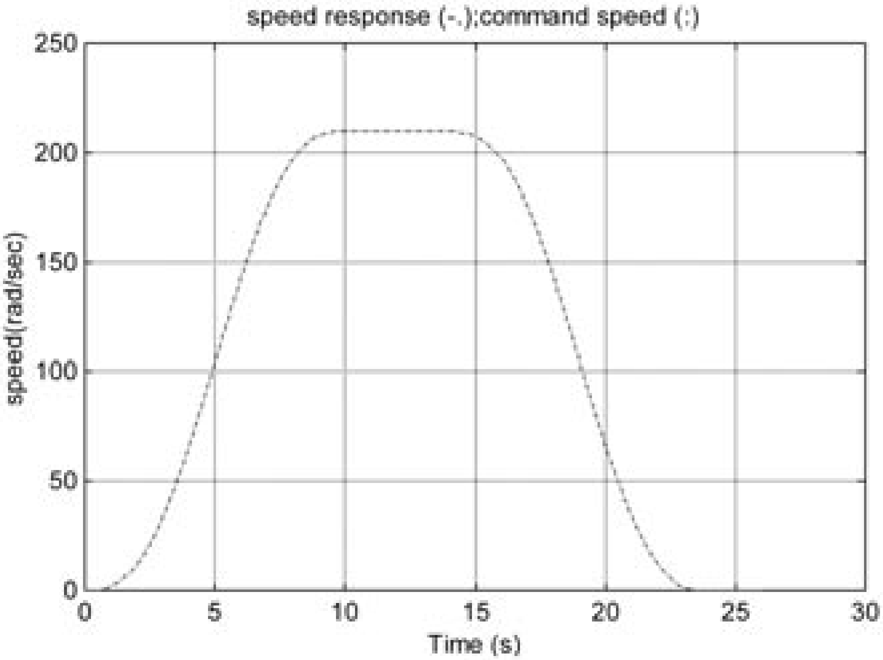
$\mathbf{P} = \begin{bmatrix}0.7059 & 0.2861\\ 0.2861 & 0.1708\end{bmatrix}$ .Simulation results for the case with EV operated on traffic condition (I) are shown in Fig. 5. From Fig. 5(a), we know that the EV speed response can track the command trajectory. The tracking error shown in Fig. 5(b) is within -7.1915×10^{-4} and 7.0355×10^{-4} rad/sec. The corresponding required motor control torque is shown in Fig. 5(c). The control torque ($T_{max}=318.3194$ N-m) is smaller than the maximum torque of the induction motor.

Simulation results for the case with EV operated on traffic condition (II) are shown in Fig. 6. From Fig. 6(a), we know that the EV speed can also follow the command trajectory. The tracking error shown in Fig. 7(b) is within -0.0023 and 0.0016 rad/sec. The corresponding required motor control torque is shown in Fig. 6(c). The control torque ($T_{max}=319.8095$ N-m) is smaller than the maximum torque of the induction motor.

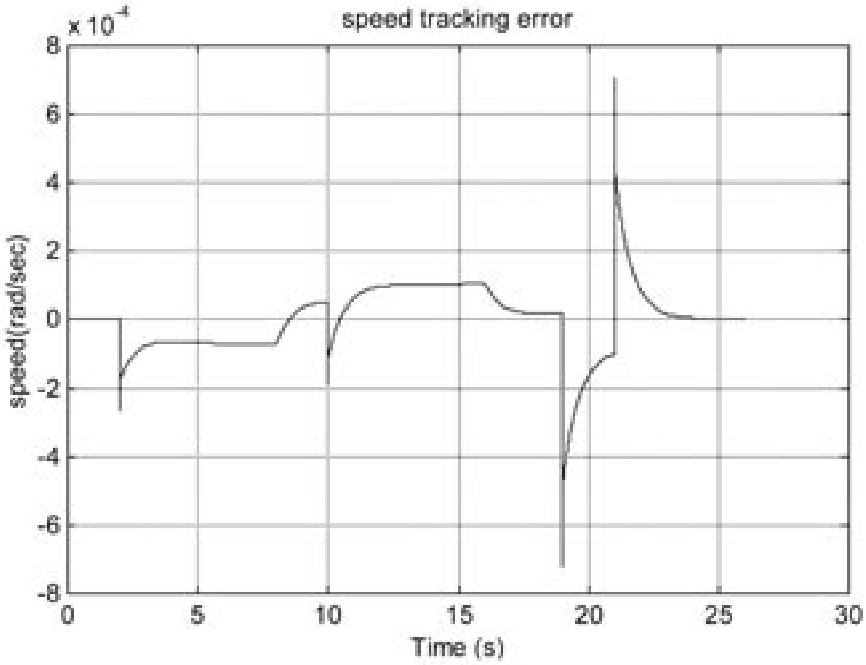
From the simulation results for this case with EV operated on the more traffic condition, we know that the speed tracking error with the LMI method is small and from Fig. 5(c), we know that control torque with the LMI method is smooth. Thus, when ρ is selected as small as possible, and the common P and the feedback gains $K_i, i = 1,2$, are found with a bigger α, So the LMI method has excellent capability to reject disturbances and high robustness with respect to uncertainty.

5. Conclusions

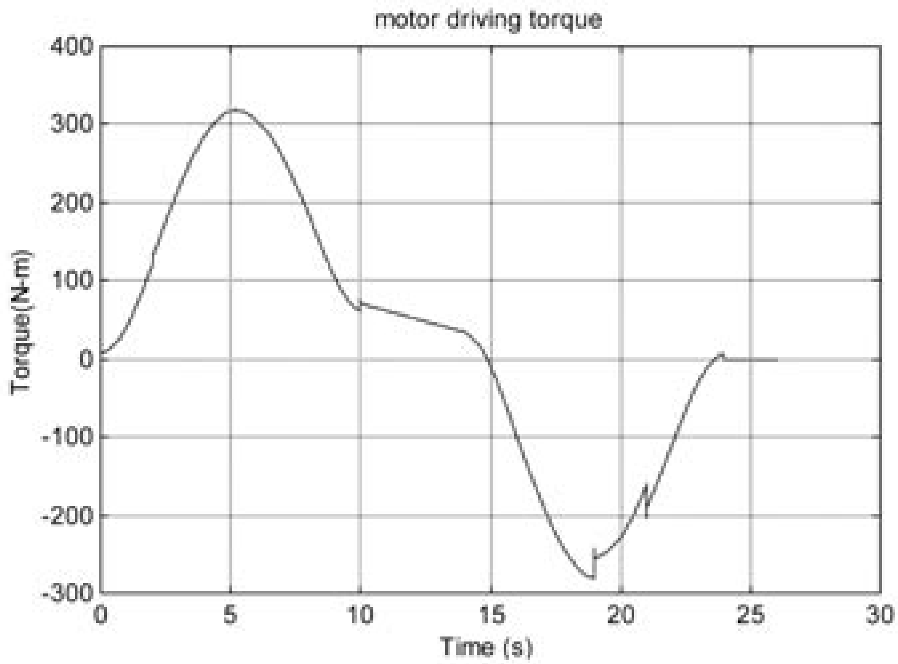
In this paper, the dynamics model and a T-S fuzzy model for EV propulsion systems with a 3-phase ac induction motor are constructed. A procedure for systematically constructing a simple T-S fuzzy model with very small number of rules that can exactly represent the EV propulsion systems with a 3-phase ac induction motor is suggested, and a PDC control design based on the T-S fuzzy model is proposed. Because the number of rules is very small, it is easy to find a common Lyapunov matrix P, and no relaxation methods are need. The feedback gains K_i and P can be simultaneously determined by considering the control design problem as a GEMP problem via LMI constraints. Proper K_i and P can be obtained by choosing the results with sufficiently high value of the Lyapunov function decay-rate scaling factor α. Simulation results are used to show that the derived LMI-based control law can guarantee the stability and robustness of an EV propulsion system when it is driven on various traffic conditions.


speed response (-.);command speed (:)
250
200
150
100
50
0
speed(rad/sec)
0
5
10
15
20
25
30
Time (s)

(a)


speed tracking error
x 10^-4
8
6
4
2
0
-2
-4
-6
-8
speed(rad/sec)
0
5
10
15
20
25
30
Time (s)

(b)



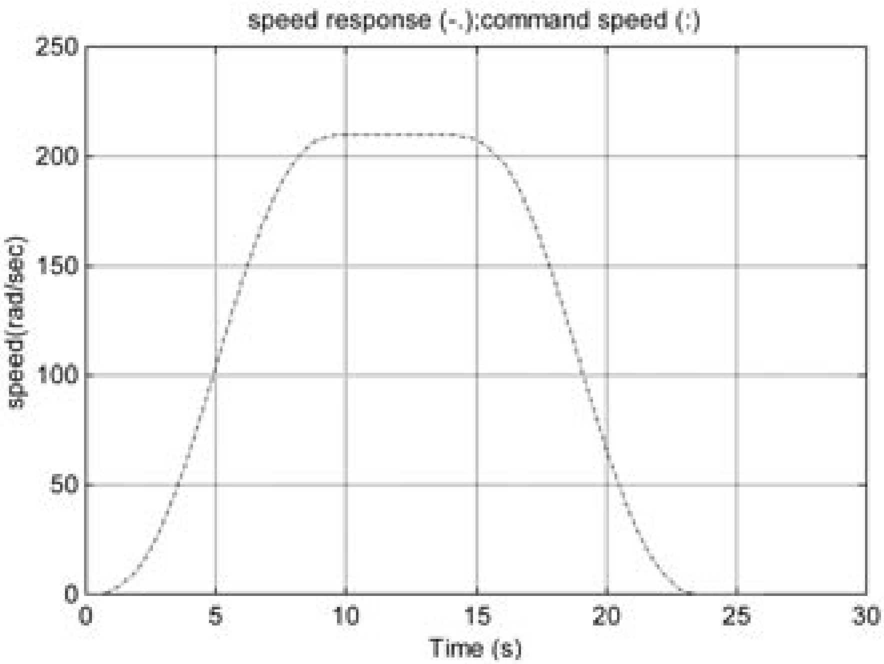
(c)

Fig 6. Simulation results for traffic condition (I) (LMI)

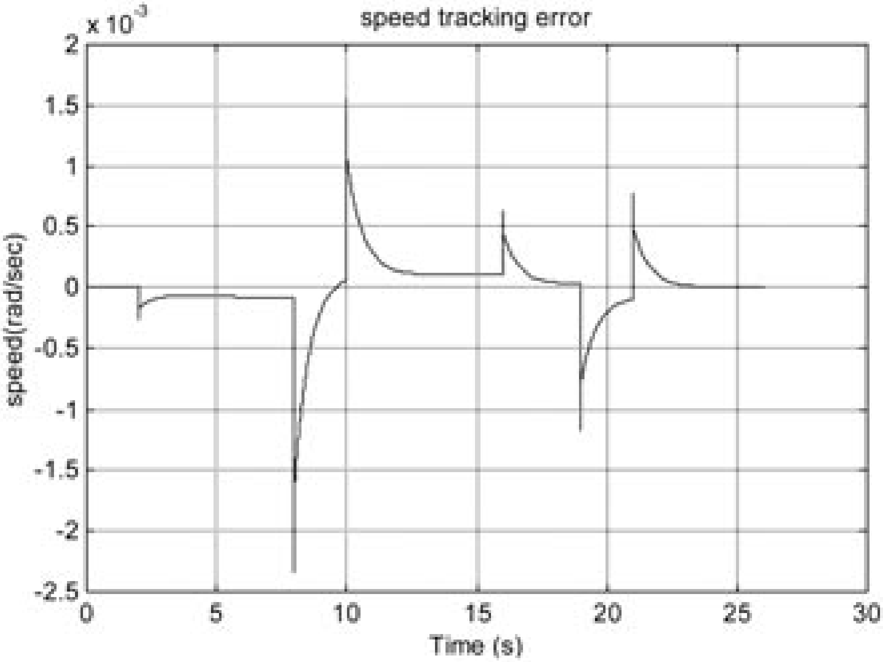
(a) Speed command and actual speed,

(b) Speed tracking error,

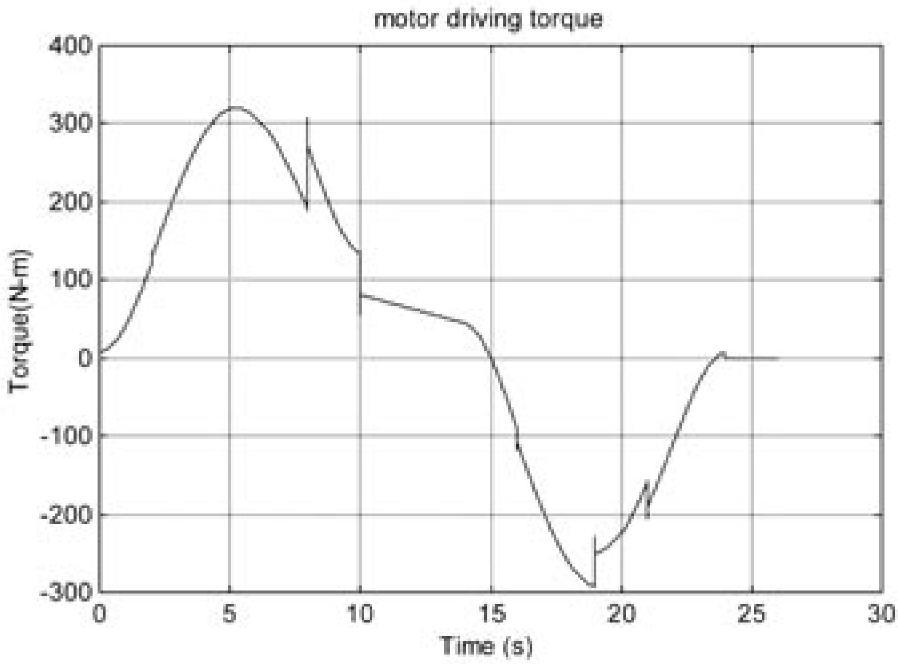
(c) Control torque.


speed response (-.);command speed (:)
250
200
150
100
50
0
speed(rad/sec)
0
5
10
15
20
25
30
Time (s)

(a)


speed tracking error
x 10^-3
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
speed(rad/sec)
0
5
10
15
20
25
30
Time (s)

(b)



(c)

Fig 7.Simulation results for traffic condition (II) (LMI)

(a) Speed command and actual speed,

(b) Speed tracking error,

(c) Control torque.

References

[1] N. Mutoh, S. Kaneko, T. Miyazaki, R. Masaki, and S. Obara, "A Torque Controller Suitable for Electric Vehicles," *IEEE Trans. Ind. Electron.*, Vol. 44, No. 1, pp. 54-63, 1997.

[2] K. K. Shyu and H. J. Shieh, "A New Switching Surface Sliding Mode Speed Control for Induction Motor Drive Systems," *IEEE Trans. on Power Electronics*,Vol. 11, No. 4, pp. 660-666, 1996.

[3] H. O. Wang, K. Tanaka and, M. F. Griffin "An Approach to Fuzzy Control of Nonlinear Systems: Stability and Design Issues," *IEEE Trans. on Fuzzy System*, Vol. 4, No. 1, pp. 14-23, 1996.

[4] Tseng, C. S., Chen, B. S., and Uang, H., J., "Fuzzy tracking control design for nonlinear dynamic system via T-S fuzzy model," *IEEE Trans. on Fuzzy Systems*, Vol. 9, No. 3, pp. 381-392, 1995.

[5] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishanan, *Linear Matrix Inequalities in System and Control Theory*, Philadelphia, PA: SIAM, 1994.

[6] M. Ehsani, K. M. Rahwan, and H. A. Toliyat, "Propulsion System Design of Electric and Hybrid Vehicles," *IEEE Trans. on Industrial Electronics*, Vol. 44, No. 1, pp. 19-27, 1997.

[7] B. K. Powell, K. E. Bailey, and S. R. Cikanek, "Dynamic Modeling and Control of Hybrid Electric Vehicle Powertrain Systems," *IEEE Control Systems Magazine*, Vol. 18, No. 5, pp. 17-33, 1998.

[8] D. W. Novotny and T. A. Lipo, *Vector Control and Dynamics of AC Drives*, Oxford 1996.

[9] B. K. Bose, *Modern Power Electronics and AC Drives*, Prentice Hall PTR, Upper Saddle River, NJ, 2002.

[10] J. J. Craig, *Introduction to Robotics*, Addison- Wesley, 1989.

[11] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis*, Wiley-Interscience, 2001.