互耦陀螺之渾沌同步

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摘要

本篇論文研究兩相同互耦陀螺之渾沌同步。在兩個處於渾沌運動的系統，利用雙向偶合方式使兩系統達成渾沌同步化。同時計算次李雅普諾夫指數且應用其正、負號來判斷渾沌同步化是否實現。研究發現當主要的次李雅普諾夫指數最後一次穿過零值時，出現渾沌同步運動。此外，亦研究渾沌同步化所需的最短時間。

關鍵字：渾沌同步、雙向耦合、次李雅普諾夫指數

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Chaos Synchronization of Mutual Coupled Gyros

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Abstract

Synchronization of the two identical chaotic motions of symmetric gyros has been studied. It has shown that one can make two identical chaotic systems to synchronize through applying dual-way coupling. The sign of the sub-Lyapunov exponents is also applied as a criterion for this. It has been found that when the last time of the major sub-Lyapunov exponent transverses the zero then chaos synchronization occurs. In addition, synchronization of chaos has been also shown by phase trajectory. Besides, the synchronization time is also examined.

Key Words：synchronization, dual-way coupling, sub-Lyapunov exponents

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1. Introduction

The symmetric gyro can be used to modal a variety of physical systems, ranging from a child's top to a modern gyroscopic navigational instrument. In 1996, Ge and his coworkers [1, 2] conducted a detailed study evaluating the nonlinear behavior of a symmetric heavy gyroscope mounted on a vibrating base. In their study, the chaotic motion of the system with linear damping was investigated. Very recently the motion of a symmetric gyro which is subjected to a harmonic vertical base excitation has been studied by Tong et al. [3], with particular emphasis on its nonlinear dynamic behavior without taking into account the damping effect. Their study has shown that a symmetric gyro exactly exhibited chaotic motion.

The possibility of two or more chaotic systems oscillating in a coherent and synchronized way is not an obvious one. One of the main features often associated with the definition of chaotic behavior is the sensitive dependent on initial conditions. Then one may conclude that synchronization is not feasible in chaotic systems because it is not possible in real systems either to reproduce exactly identical initial conditions or exact specification of system parameters for two similar systems. But, the recent suggestion of Pecora et al. [4] that it is possible to synchronize even chaotic systems by introducing appropriate coupling between them has revolutionized our understanding. They have shown that if a response (slave) system responses to a chaotic signal from the drive system, under some conditions the signal in the response would converge to the corresponding signals in the drive (master) system. Such a possibility is known as synchronization of chaotic systems. Synchronization can be thought of as a form of control chaos. Since their work, Synchronization of chaotic dynamical systems has been intensively studied [5]. Very recently, Chen and Lin [6] have shown that two identical chaotic symmetric gyros can synchronize through applying one-way coupling. In this paper, it will be studied that if two identical chaotic symmetric gyros can synchronize through applying dual-way coupling or not. The sign of the sub-Lyapunov exponent will be applied to diagnose synchronization of chaos occurs or not. Synchronization of chaos will be also shown by phase trajectory. Further, the synchronization time will be also examined.

2. Equations of Motion

The geometry of the problem under consideration is depicted in Fig. 1. The motion of a symmetric gyro mounted on a vibrating base can be described by Euler's angles ((nutation), ((precession), and ((spin). By Lagrangian approach that the Lagrangian has the expression

$$L = \frac{1}{2} I_1 (\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta) + \frac{1}{2} I_3 (\dot{\phi}\cos\theta + \dot{\psi})^2 - Mg(\ell + \bar{\ell}\sin\omega t)\cos\theta \quad (1)$$

where I_1 and I_3 are the polar and equatorial moments of inertia of the symmetric gyro, respectively, Mg is the gravity force, is the amplitude of the external excitation disturbance, and is the frequency of the external excitation disturbance. It is not difficult to see that coordinates ϕ and Ψ are cyclic, the momentum integrals are

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = I_1 \dot{\phi} \sin^2\theta + I_3 (\dot{\phi}\cos\theta + \dot{\psi})\cos\theta = \beta_\phi \quad (2)$$

$$P_\psi = \frac{\partial L}{\partial \dot{\psi}} = I_3 (\dot{\phi}\cos\theta + \dot{\psi}) = I_3 \omega_z = \beta_\psi \quad (3)$$

where ωz is the spin velocity of the gyro. The dissipation function is also given by

$$F = \frac{1}{2} C \dot{\theta}^2 \quad (4)$$

where C is a positive constant. Applying Routh's procedure and the above relation, the equation governing the gyro is given by

$$\ddot{\theta} + \frac{\beta_\phi^2}{I_1^2} \frac{(1-\cos\theta)^2}{\sin^3\theta} + \frac{C}{I_1}\dot{\theta} - \frac{Mg\ell}{I_1}\sin\theta = \frac{Mg\bar{\ell}}{I_1}\sin\omega t \sin\theta \quad (5)$$

The normalized equations in convenient first order form are

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - cx_2 + \beta \sin x_1 + f \sin \omega t \sin x_1 \quad (6)$$

where

$$x_1 = \theta, x_2 = \dot{\theta}, \alpha = \frac{\beta_\phi}{I_1} = \frac{I_3 \omega_z}{I_1},$$
$$c_1 = \frac{C}{I_1}, \beta = \frac{Mg\ell}{I_1}, f = \frac{Mg\overline{\ell}}{I_1} \quad (7)$$

The detailed explanation for the equations of motion the reader is referred to reference [1, 2].

3. Chaos Synchronization by Dual-Way Coupling

Chaos synchronization is an important problem in the nonlinear science. Chaos synchronization problem has the following feature: The trajectories of a slave (response) system must tracks the trajectories of the master (drive) system in spite of both master and slave system being different. In this section, we will study that if two identical chaotic symmetric gyros can synchronize through applying dual-way coupling or not. The state equations of two identical gyros with dual-way coupling element are represented as

drive(master):

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -\alpha^2 \frac{(1-\cos x_1)^2}{\sin^3 x_1} - cx_2 + \beta \sin x_1 + f \sin \omega t \sin x_1 + \varepsilon G(x_2, y_2) \quad (8)$$

response(slave):

$$\dot{y}_1 = y_2,$$
$$\dot{y}_2 = -\alpha^2 \frac{(1-\cos y_1)^2}{\sin^3 y_1} - cy_2 + \beta \sin y_1 + f \sin \omega t \sin y_1 + \varepsilon F(x_2, y_2) \quad (9)$$

where ε is the coupling parameter, $F(x_2, y_2)= \sin(x_2-y_2)$ and $G(x_2, y_2)= \varepsilon \sin(y_2-x_2)$ are the coupling functions. It is nature, when the one-way coupling (as function G is not presented) applying one knows that the behavior of the slave system is dependent on the behavior of the master system, but the second one is not influenced by the behavior of the first. Clearly, the behaviors of the slave system and master system are interacted owing to dual-way coupling. In fact, synchronization of chaos can be still regarded as a special tracking problem, with the desired trajectory not being a constant. In addition, as the dual-way coupling is applied, it is known that desired strange attractor could be changed. The dual-way coupling is also a control law. The above coupling terms possess the same character, when the synchronization occurs ($x_i(t)=y_i(t)$, i=1,2) then them vanish.

The system exhibits chaotic behavior under the parameter condition：α^2=100, β=1, c=0.5, ω= 2, f=34.5. Fig. 2-3 show phase trajectory and Poincaré map for f=34.5. An interesting strange attractor, which configuration looks like "S", is presented. The detailed analysis and discussion the reader is referred to reference [1, 2].

In order to study synchronization of chaos, the influence of the coupling strength on the two identical systems behavior will be examined by observing how the coupling parameter ε changes with the constant values of the remaining parameters (, (=1, c=0.5, = 2, f=34.5). The initial conditions of master system and slave system are also given as (1.0, 0.2) and (0.1, 0.02), respectively. In the present study, the equations (8) and (9) will be integrated numerically against ε ([0,1.0], while the incremental value of ε is 0.01 in order to obtain the Lyapunov exponents. For the whole system (equations (8) and (9)), the Lyapunov exponents are presented as (λ1, λ2, λ3, λ4,0). Where λ3 and λ4 are called sub-Lyapunov exponents. As function G is not given, the master system exhibits chaotic motion under the parameter condition: α2=100, β=1, c=0.5, ω= 2, f=34.5. In this situation, the Lyapunov exponents λ1 and λ2 are always presented as (λ1>0, λ2<0) for any coupling strength ε. When the dual-way coupling introduced, the Lyapunov exponents λ1 and λ2 will be changed depending on the coupling strength ε. For the whole system the Lyapunov exponent types can be classified as: (I)(+，－，+，－，0), hyperchaos, (II) (+，－，－，－，0), chaos, and (III) (－，－，－，－，0), regular motion. The sub-Lyapunov exponent types are presented as (+, -) and (-, -). The Lyapunov exponents of the whole system against ε are shown in Fig. 4.

We know that the master system and the slave system will synchronize only if the sub-Lyapunov exponents are all negative. And the above theorem is a necessary, but not sufficient condition for synchronization. From above analysis, one finds that only type (II) could perform chaos synchronization.

Fig. 4 shows an interesting phenomenon, the major sub-Lyapunov exponent transverses zero many times in certain interval of ε. It is obvious that the detailed discussion is needed. It is found that the sub-Lyapunov exponents are all negative at $\varepsilon \in [0.15, 0.19]$ and ε 0.24. While the Lyapunov exponent type is (—，—，—，—，0) at $\varepsilon \in [0.16, 0.18]$, the master and slave system exhibit regular motion. The master system and slave system are calculated by numerical integration against ε =0.15, 019 ε 0.24, while the incremental value of ε is 0.01. The fascinating behavior of the slave system occurs. The attractor of the slave system is changed into another configuration for ε =0.15 and ε =0.19, respectively. It is obvious that synchronization will not present. Finally, if ε 0.24, the phase portraits of master and slave are synchronized. Fig. 5(a) depicts the trajectory of (x2-y2), for ε =0.30. Fig. 5(b) shows the relation of x2 and y2 as synchronization occurs. Fig. 5(c) and (d) display the trajectories of master system and slave system, respectively. On the contrary, if ε =0.2, the two identical systems will not synchronize as shown in Fig. 6.

From the previous analysis, we can conclude that the coupling strength ε plays a major role to make two identical chaotic systems synchronizing. Besides, it is found that when the last time of the major sub-Lyapunov exponent transverses the zero then chaos synchronization occurs.

4. Synchronization Time

In the previous study, the dual-way coupling has been successfully applied to perform chaos synchronization in the two identical systems. Next, we address the following question: when can one make a chaotic trajectory of one system to synchronize with a chaotic trajectory of the other system with dual-way coupling. For this purpose, we definite an error function

$$E(t) = |x_1 - y_1| + |x_2 - y_2| + |\dot{x}_1 - \dot{y}_1| + |\dot{x}_2 - \dot{y}_2| \qquad (10)$$

When the value of function E(t) is less than 10-6, then the two identical systems synchronizing are archived. At that time t, which is called "synchronization time". According the above rigorous definition, if ε =0.30 the synchronization time is 383.6 seconds.

5. Conclusions

Synchronization of the two identical chaotic motions of symmetric gyros has been studied. It has shown that one can make two identical chaotic systems to synchronize through applying dual-way coupling. The sign of the sub-Lyapunov exponent has been applied to diagnose synchronization of chaos occurs or not. It has been also found that when the last time of the major sub-Lyapunov exponent transverses the zero then chaos synchronization occurs. In addition, synchronization of chaos has been also shown by phase trajectory. Besides, the synchronization time is also examined.

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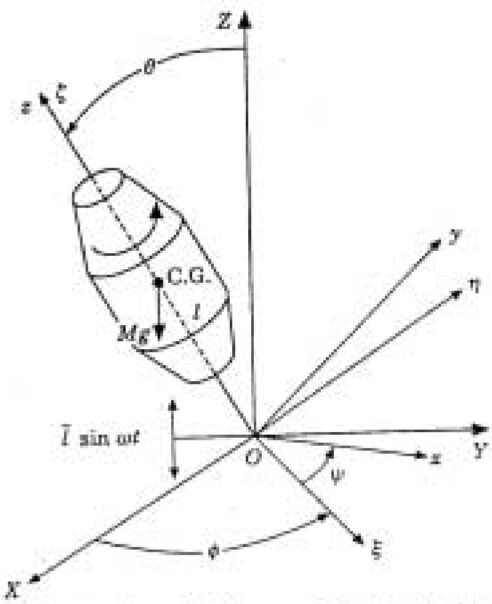
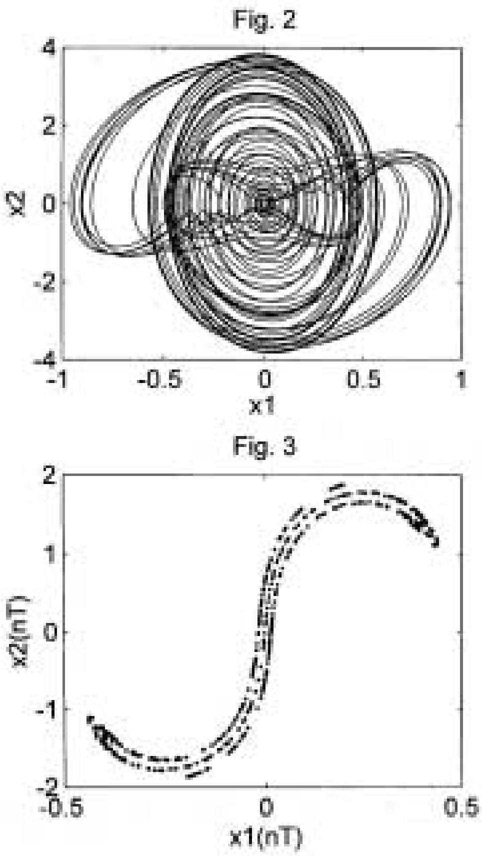


Fig. 1. A schematic diagram of the physical system.



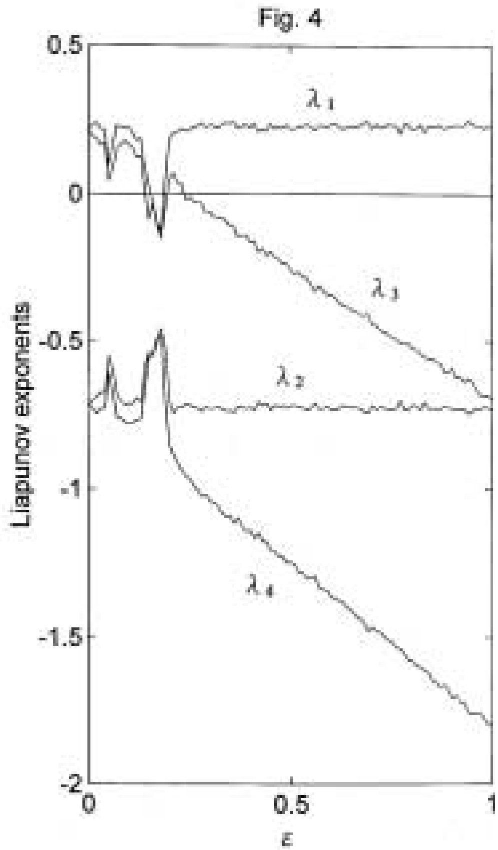
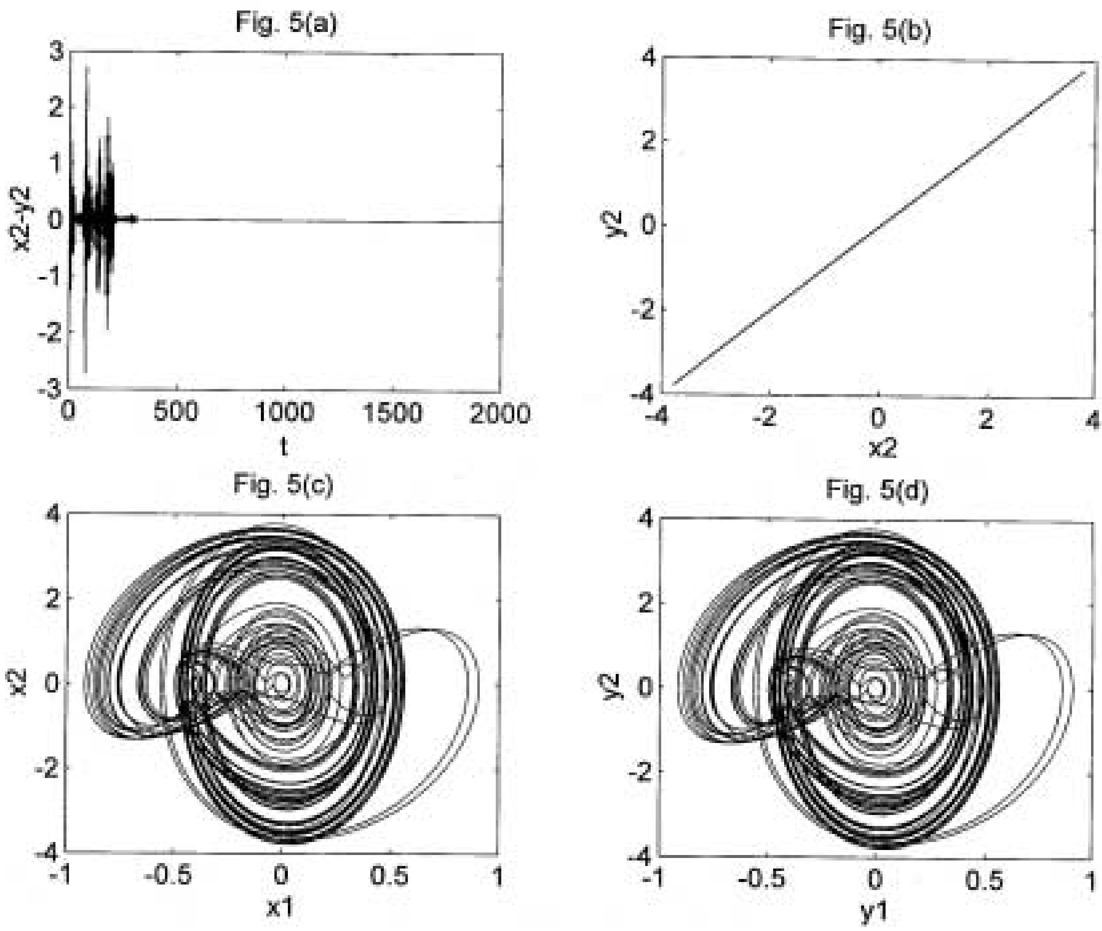


Fig. 2-3 The Phase trajectory and the Poincaré map of the system for specific values set (α^2=100, β=1 c_1=0.5, c_2=0.05, ω=2, f=34.5).

Fig. 4 The Liapunov exponents of mutual coupled gyros versus ε .



5 (a)The trajectory of (x_2-y_2), (b) The relation of x_2 and y_2, (c) The trajectories of the master system, (d) The trajectories of the slave system (Synchronized motion for ε =0.3).

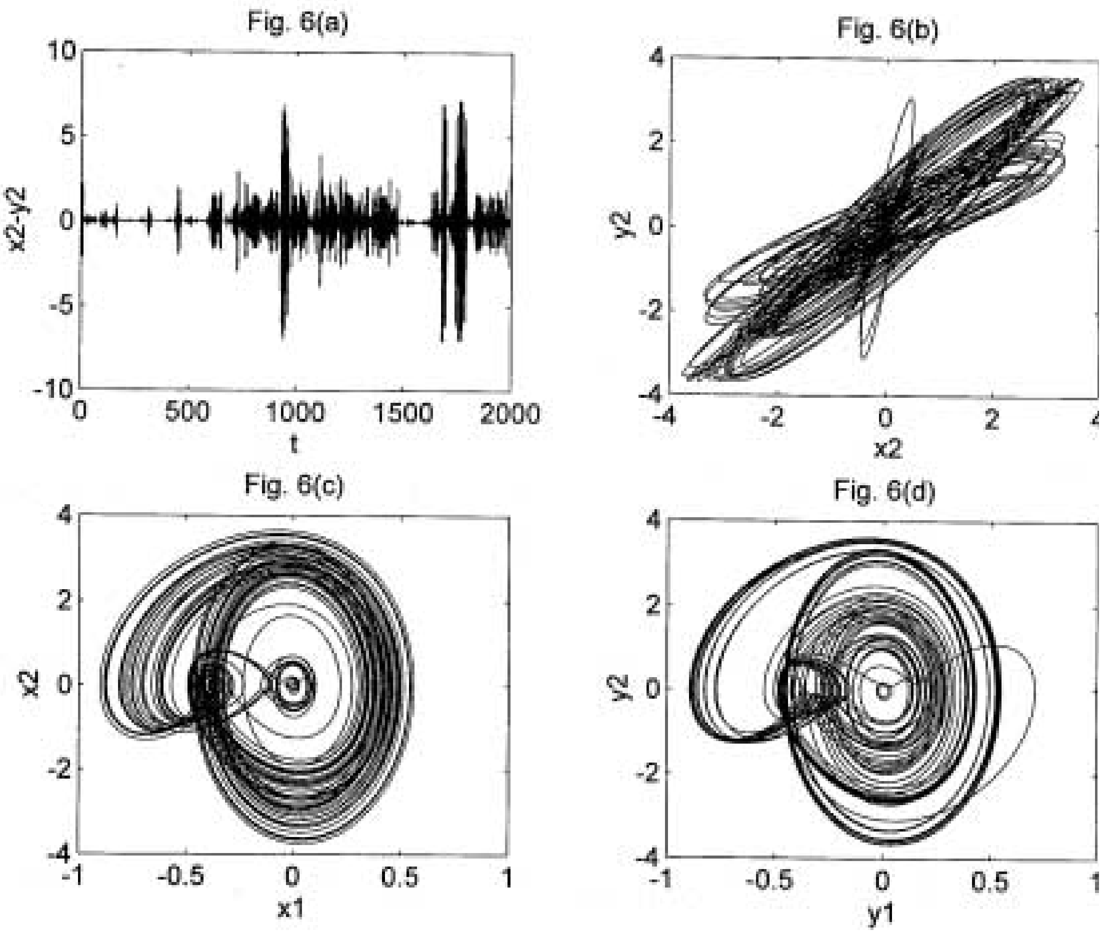


Fig. 6 (a) The trajectory of (x_2-y_2), (b) The relation of x_2 and y_2, (c) The trajectories of the master system, (d) The trajectories of the slave system (Unsynchronized motion for ε =0.2).