# Influence of Heat Transfer and Stretch on Excess Enthalpy Burning

Chih-Hsin Tsai

## Abstract

A steady, one-dimensional pemixed flame propagating in a duct with varying cross-sectional area with external heat recirculation, is analyzed using activation energy asymptotics. Heat recirculation is achieved by transferring heat through a tube wall within a given distance L. The external heat transfer results in globally external heat loss and excess enthalpy burning (which is globally adiabatic), respectively, to the system with increasing wall temperature. The influences of external heat recirculation on the flammability limit and extinction of a premixed flame are examined with three parameters, namely, the amount of external heat transfer, the flame stretch and the external heat transfer coefficient.

**Key words**: Excess enthalpy burning, Flammability limit, Flame stretch, Lewis number

Chih-Hsin Tsai, Associate Professor, Department of Mechanical Engineering, HIT.

# 熱傳和火焰拉伸對超焓燃燒的影響

蔡志信

## 摘 要

超焓燃燒主要是利用熱交換原理，將火焰下游高溫產物的熱量傳遞給上游未燃混合氣，使未燃氣在通過火焰之前先經預熱，以達到超焓的目的。本研究利用高活化能極限近似微擾理論來探討具橫截面積改變的一維流場中之層流、穩態、預混火焰具外部熱迴流的燃燒特性。期望瞭解因橫截面積變化所引起之火焰拉伸、外部熱傳導係數和外部熱傳量對可燃極限和火焰熄滅的影響。

**關鍵詞:** 超焓火焰，可燃極限，火焰拉伸，路易數(Le數)

蔡志信：修平技術學院機械工程系副教授

## Introduction

The concept of excess enthalpy burning to burn mixtures of very low heat content was first proposed by Weinberg [1], and further developed by a series of theoretical and experimental studies [2-4]. The burning of an excess enthalpy flame was achieved by transferring the lost energy from products downstream of a flame to preheat the fresh mixture upstream through external heat recirculation. It was generally concluded that heat recirculation improves the flame stability and extends the flammability limit. In 1979, Takeno and Sato [5] proposed a way of producing an excess enthalpy flame by inserting a semi-infinite porous solid of high thermal conductivity into the one-dimensional flame zone. Deshaies and Joulin [6] presented an analytical study using matched asymptotics in the limit of large activation energy which performed in the case studied by Takeno and Sato. They confirmed the capability of Takeno and Sato’ s devise to allow an increase of the reaction temperature above the theoretical adiabatic flame temperature, such that a widening of the flammability domain was anticipated.

Takeno et al. [7] further used a finite solid length instead of an infinite one [5] for internal heat recirculation. They concluded that there exists a critical mass flow rate above which the combustion cannot sustained, and the critical flow rate is more than ten times as large as the normal flame. They also found that below the critical mass flow rate the system has two possible combustion states with the distinct solid temperature. Buckmaster and Takeno [8] suggested the possibility of flame stabilization in, ahead of, and behind the porous solid, depending on controllable parameters. Moreover, global heat losses were allowed and the phenomena of blow-off and flashback were reported in the study. A further theoretical study was made by Takeno and Hase [9] to examine the effects of the solid length and the heat loss on the excess enthalpy flame. They concluded that an increase in the solid length results in an increase in the critical maximum flow rate above which combustion cannot be sustained, and that an increase in the heat loss causes a reduction in flammability domain.

Some experimental studies [10-12] have been reported that the observed flame stability and combustion characteristics were in accordance with theoretical predictions. These flames , far below the normal flammability limit, could sustain stable burning up to very high flow rate. Additionally, the emission characteristics of $NO_x$ and CO were found to be well controlled by Hashimoto et al. [11].

Studies on excess enthalpy burning introduced above were only focused on homogeneous mixture propagating in a duct of constant area. Flame stretch is recognized as a very important parameter affecting flame behavior. The effects of stretch become especially prominent in the presence of preferential diffusion, when the mixture has nonunity Lewis number [13-15]. For nonequidiffusive, stretched flame, the flame response exhibits opposite behavior when the stretch is positive or negative, and when the mixture's Lewis number is greater or less than unity. For positive stretched flames in the stagnation-point flow, increasing stretch weaken/extinguishes a Le>1 flame but intensifies a Le<1 flame [13-15]. The converse holds for the negatively stretched Bunsen flame tip [16,17].

Additionally, the effects of area change in one-dimensional formulation do play a significant role in flame behavior [18]. There was also a study for the propagation of a premixed flame in a close tube with varying cross-section area [19]. It was concluded that positive flame stretch increase the mass burning rate, negative stretch has the opposite effect for flames with a Lewis number larger (smaller) than one. Therefore, the objective of this study is to analyze the influences of flame stretch, preferential diffusion and external heat loss on excess enthalpy flames propagating in a duct with varying cross-sectional area using activation energy asymptotics.

## Theoretical Model

As schematically illustrated in Fig. 1, we adopt a one-dimensional coordinate system in which a planar flame sits at $x = x_f$ in a duct with varying cross-sectional area, the premixed combustible mixture comes from $x = -\infty$ ; and equilibrium reaction products move away toward $x = +\infty$ . We further assume that the external heat recirculation is achieved by transferring heat through a tube wall maintained at a constant temperature, $T_S$ , within a given distance from 0 to L. Since the external heat transfer is small compared with the heat release of combustion, it is reasonable to assume that the amount of external heat transfer is of $O(\varepsilon)$ in the asymptotic analysis. Here $\varepsilon = T_\infty / T_a$ is the small parameter of expansion for large activation energy reactions in the combustion process.

Finally, we assume that the fuel and oxidizer reaction for the bulk premixed flame is one-step overall, and the conventional constant property simplifications apply.

The present case for a duct with varying cross-sectional area can be modeled by adding $-\rho u$, $(1/Le)(dY/dx)$, $dT/dx$ times $(1/A)(dA/dx)$ [20] to the right-hand sides of the non-dimensional equations for gas-phase continuity, conservation of fuel, oxidizer, and energy. The function A denotes the cross-sectional area of the duct, which is chosen to be a slowly varying function of $x$. Therefore, $(1/A)(dA/dx)$ is of $O(\varepsilon)$ in the asymptotic analysis. Accordingly, these equations are, respectively, given by,

$$\frac{d}{dx}(\rho u)=(-\rho u)(\frac{1}{A}\frac{dA}{dx}) \tag{1}$$

$$\frac{d}{dx}(\rho uY_F-\frac{1}{Le}\frac{dY_F}{dx})=\dot{W}+\frac{1}{Le}\frac{dY_F}{dx}(\frac{1}{A}\frac{dA}{dx}) \tag{2}$$

$$\frac{d}{dx}(\rho uY_O-\frac{1}{Le}\frac{dY_O}{dx})=\dot{W}+\frac{1}{Le}\frac{dY_O}{dx}(\frac{1}{A}\frac{dA}{dx}) \tag{3}$$

$$\frac{d}{dx}(\rho uT-\frac{dT}{dx})=-\dot{W}-\varepsilon K(T-T_s)H(x)+\frac{dT}{dx}(\frac{1}{A}\frac{dA}{dx}) \tag{4}$$

where

$$\dot{W}=-(\frac{B'\sigma}{\tilde{\bar{M}}'_O})(\frac{P'\tilde{\bar{M}}'}{\tilde{R}})^2(\frac{\lambda'}{C'_{PG}\dot{m}'^2_P})Y_OY_F\exp(-\frac{T_a}{T}) \tag{5}$$

and the function $H(x)$ in eq.(4) is equal to 1 as $0\le x\le L$ or 0 as $x>L$, while $x=x'/l'_T$ is the non-dimensional distance expressed in units of the preheat zone thickness, $l'_T=\lambda'/(C'_{PG}\dot{m}'_P)$. During the derivation, $(1/A)(dA/dx)$ has been stretched as $\varepsilon\Gamma$ [19]. Here $\Gamma$ is called the stretch parameter. A positive value of $\Gamma$ represents that the flame propagates in a divergent section, while a negative value of $\Gamma$ denotes that the flame propagates in a convergent section. $K$ represents the external heat transfer coefficient.

Performing the inner and outer expansions based on the small parameter of $\varepsilon$, and following the detailed matching procedure of the previous study [21] to match the inner and outer solutions, we therefore have the final results as follows:

$$\dot{m}_0^2=\exp[T_1^+(x_f)] \tag{6}$$

when $0\le x_f\le L$ Equation (6) indicates that the flame propagation flux is exponentially affected by the first-order temperature downstream near the flame. The first-order temperature $T_1^+(x_f)$ is expressed by the following equation:

$$\begin{aligned}T_1^+(x_f)=&\frac{K}{\dot{m}_0^3T_\infty L}(T_\infty-T_{-\infty})(1-e^{-\dot{m}_0x_f}-\dot{m}_0x_f)\\&\cdot(1-e^{\dot{m}_0(x_f-L)}-\dot{m}_0L+\dot{m}_0x_f)\\&-\frac{KQ}{\dot{m}_0^3T_\infty L}(1-e^{\dot{m}_0(x_f-L)}+\dot{m}_0x_f)\\&-\frac{\Gamma}{\dot{m}_0T_\infty}(T_\infty-T_{-\infty})(1-\frac{1}{Le})\end{aligned} \tag{7}$$

A measure of the overall external heat transfer is given by

$$\int_0^L (T - T_s)dx = Q \qquad (8)$$

where $Q = 0$ and $Q > 0$ correspond to the system being globally adiabatic and experiencing external heat loss, respectively.

On the basis of the formulated results, Eq. (7), sample calculations for propane burning in air are now considered to illustrate the flame characteristics and the flammability limits under the external heat recirculation.

## Adiabatic Excess-Enthalpy Flame ( $Q = 0$ )

By assuming $Q = 0$, $L = 2$, and $K = 5$, we first investigate the variations of the flame flux $\dot{m}_0$ and the flame position $x_f$ as a function of flame stretch $\Gamma$ for a rich propane flame ( $\Phi_G = 2.0$, Le=0.963). In Fig. 2, it is found that the lower branch correspond to the stable solutions, the flame moves further downstream with increasing flame flux. The upper branch shows an opposite trend; therefore, it is unstable and unexistent. The upper and lower branches are conjoined at the critical point represented by the symbol ●, which indicates that there is a maximum flame flux ( $\dot{m}_B$ ), above which the flame will blow off and no combustion can be sustained. Furthermore, the decrease of $\dot{m}_0$ lower than 1 in the lower branch leads to flame flashback ( $x_f < 0$ ). In the region of positive stretch, the increase of flame stretch results in the increase in the flame flux for a Le<1 flame. However, when the flame experiences negative stretch, it reveals an opposite trend. This mainly results from the strengthening/weakening the burning intensity by flame stretch when a flame experienced positive/negative stretch for a Le<1 flame. This explains that the lower branches moves to the region of lower $\dot{m}_0$ with decreasing $\Gamma$. Furthermore, the flammability limit can be identified by the extent between the maximum and minimum flame fluxes of the lower branch in Fig. 2. Results show that the flammability limit of a positively-stretched flame is higher than that of a negatively-stretched flame for a Le<1 flame. This is because the flame-strengthening effect of positive stretch for a Le<1 flame.

Considering the influences of the external heat transfer coefficient $K$ is shown in Fig. 3 and 4. Irrespective of whether a Le<1 flame experiences positive or negative stretch, the burning intensity is enhanced with increasing the external heat transfer coefficient $K$. This is because the larger $K$ can transfer more heat from the product of downstream to preheat the fresh mixture of upstream, and thus the flammability limit domain is extend.

Figure 5 shows the variations of the flame flux $\dot{m}_0$ and the flame position $x_f$ as a function of flame stretch $\Gamma$ for a lean propane flame ($\Phi_G = 0.8$, Le=1.78). The effects of $\Gamma$ on flame flux for a lean propane flame are contrary to those of a rich propane flame. The burning intensity of a Le>1 flame is enhanced by negative stretch but diminished by positive stretch. This results from the Lewis number effect. The maximum flame flux ($\dot{m}_B$) for a negatively-stretched flame is greater than that of a positively-stretched flame at a fixed value of $K$. This is because the negative stretch would strengthen the burning intensity.

Figure 6 and 7 show the effects of external heat transfer coefficient $K$ on flame flux $\dot{m}_0$ for a lean propane flame with positive stretch and negative stretch, respectively. It is shown that the flammability limit is increased with increasing the value of $K$ irrespective of whether the stretch is positive or negative.

**Non-Adiabatic Excess-Enthalpy Flame ($Q > 0$)**

In order to understand the influence of the external heat loss ($Q$>0) on the excess enthalpy flame, the variations of the flame position as a function of the flame flux with different values of $Q$ for a flame with positive stretch $\Gamma = 3$ ($\Gamma = 1$) and negative stretch $\Gamma = -3$ ($\Gamma = -1$) are shown in Fig. 8 (Fig. 10) and Fig. 9 (Fig. 11), respectively, for rich (lean) flame of $\Phi_G = 2.0$ ($\Phi_G = 0.8$).

For the case of a small amount of external heat loss ($Q = 0.005$ and $0.007$ in Fig. 8, $Q = 0.003$ and $0.006$ in Fig. 9, $Q = 0.001$ and $0.003$ in Fig. 10, $Q = 0.007$ and $0.011$ in Fig. 11), the curves are controlled by blow-off and flashback. As the external heat loss is increased, it is found that the critical flame flux at flashback is reduced, and that the critical value of $x_f$ at blow-off is decreased, resulting in a reduction in the possible combustion region. However, a further increase in the external heat loss will lead to that the curves governed by blow-off and flashback are changed to C-shaped curves. The right and the left branches of the extinction curve normally represent the stable and unstable solutions, respectively, and are conjoined at critical points denoted by E in the figures. For the C-shaped extinction curve, Figs. 8 through 11 clearly reveal that the flame position on extinction is promptly decreased with increasing the amount of external heat loss but the corresponding flame flux is almost the same. The more the external heat loss, the greater is the effect on flame weakening.

## Conclusions

In this asymptotic analysis, an excess enthalpy theory was developed to explore the influences of external heat recirculation, flame stretch and Lewis number on the flammability limit and flame extinction of premixed propane/air flame. The positive (or negative) stretch coupled with Lewis number (Le) weakens (or increase) the burning intensity of the lean propane/air flame with Le>1 but intensifies (or decrease) the burning intensity of the rich propane/air flame with Le<1. For Le<1 (Le>1), the flammability limit of a positively-stretched flame is wider (narrower) than that of a negatively-stretched flame.

Note that the extent of flammability controlled by blow-off and flashback is decreased with increased external heat loss for lean and rich flames. However, extinction curves show that the flame position on extinction is decreased by increasing external heat loss, but the corresponding flame flux remains almost the same for both lean and rich flames.

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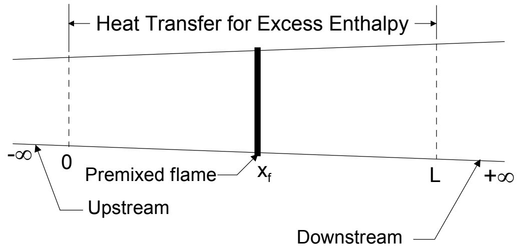


Fig. 1 Schematic diagram of an excess enthalpy flame.

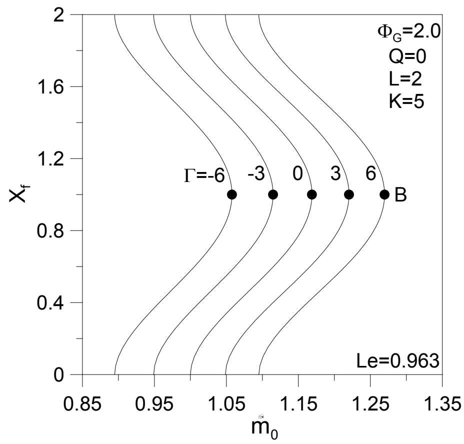


Fig. 2 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of flame stretch for a rich flame.

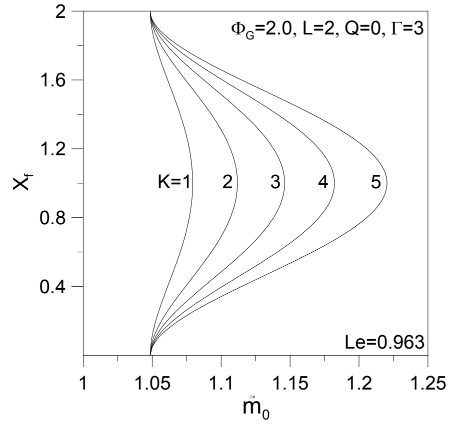


Fig. 3 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of external heat transfer coefficient for a rich flame experiencing positive stretch.

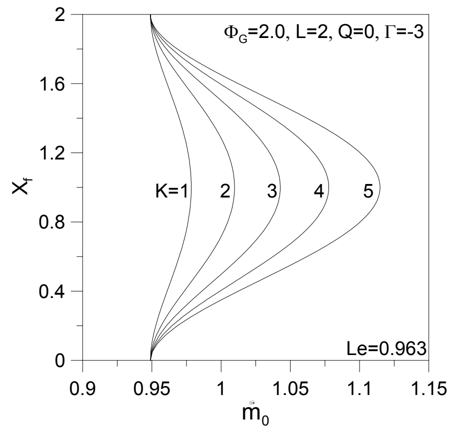


Fig. 4 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of external heat transfer coefficient for a rich flame experiencing negative stretch.

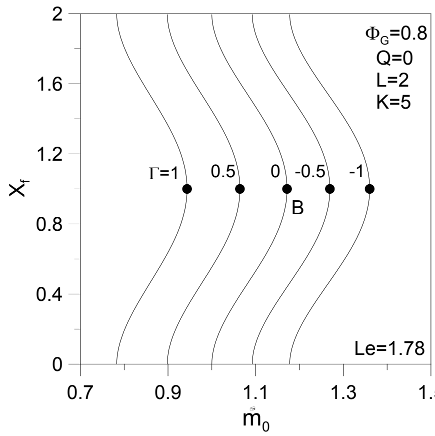


Fig. 5 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of flame stretch for a lean flame.

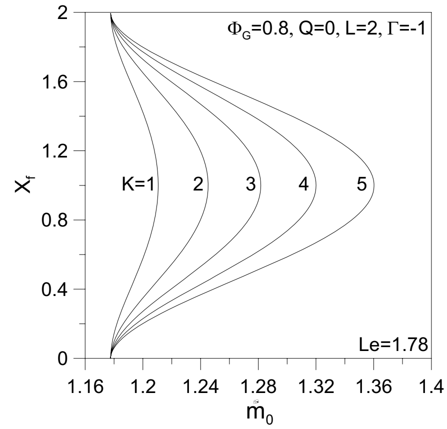


Fig. 7 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of external heat transfer coefficient for a lean flame experiencing negative stretch.

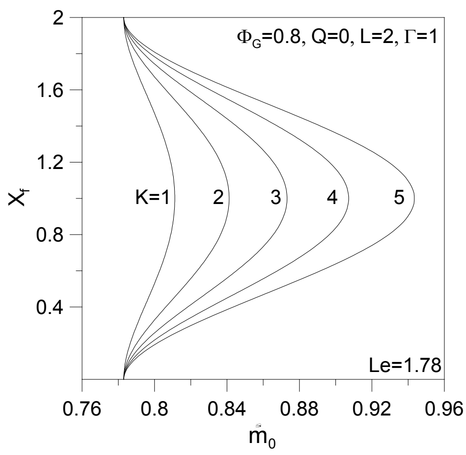


Fig. 6 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ under excess enthalpy burning with various values of external heat transfer coefficient for a lean flame experiencing positive stretch.

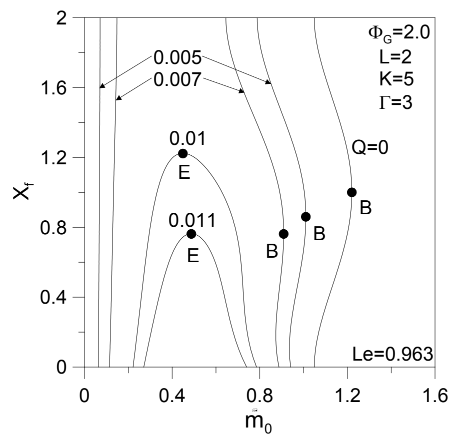


Fig. 8 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ with various amount of external heat loss for a rich flame experiencing positive stretch.

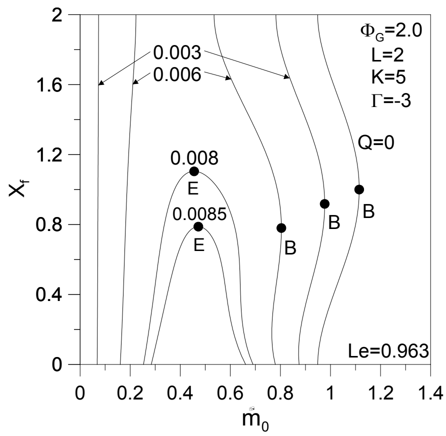


Fig. 9 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ with various amount of external heat loss for a rich flame experiencing negative stretch.

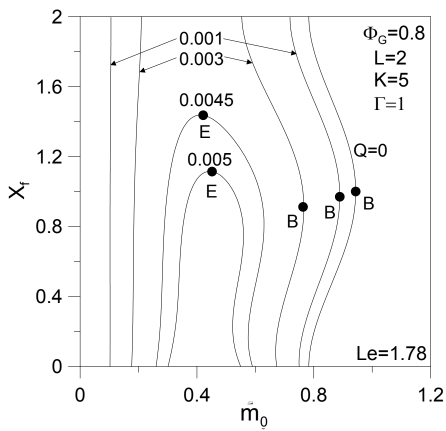


Fig. 10 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ with various amount of external heat loss for a lean flame experiencing positive stretch.

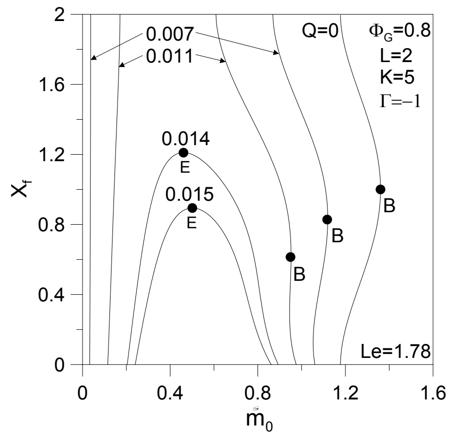


Fig. 11 Flame position $x_f$ as a function of flame flux $\dot{m}_0$ with various amount of external heat loss for a lean flame experiencing negative stretch.