# Production-Distribution and Replenishment Strategies for Deterioration Items in Supply Chain Network

Shao-Chang Miao

## Abstract

This study investigates the optimal production-distribution and replenishment (PDR) strategies of a deteriorating item in the supply chain network (SCN) system. This paper applied heuristic algorithm and integration approach. Through preliminary evaluation of the model and the solution method, the analysis shows that optimal PDR strategies bring benefits in terms of the production-inventory cost savings to firms. Computational results also point out how managers may decide on the production and distribution periods for an improved benefit realization through the managerial implications in the SCN system. Its advantages are using simple calculation and modeling process. To obtain the production period, stationary production/distribution, order quantity and average total cost, the manager only needs to input the time-varying inventory levels.

**Keywords:** Deteriorating item; Supply chain network (SCN); Production-Distribution Replenishment (PDR) strategy; Heuristic algorithm; Integration approach.

---

Shao-Chang Miao, Office of Military Education, HIT. Doctoral program student, Department of Statistics, TungHai University.
投稿日期：980902 接受刊登日期：981202

# 退化性商品在供應鏈網路的生產配銷與補貨策略

繆紹昌

## 摘 要

本研究在探討一個退化性的商品在供應鏈網路系統中最佳的生產、配銷和補貨的策略。我們運用啓發式演算法和積分法等兩項方法，經由模型和解題方法的初步評估，這個分析顯示最佳的生產、配銷和補貨策略對公司的生產與存貨成本的節約而言帶來了好處，計算的結果也指出：管理者在供應鏈網路系統中爲了增進利益的實現下，所需決定的生產和配銷週期於經營管理上的涵意。它的優點是採用簡單的演算與不複雜的模式化，管理者只需輸入在觀察點的存貨量，便可求得生產週期、穩定的生產配銷、訂購量與平均總成本。

**關鍵詞：**退化性商品、供應鏈網路、生產配銷與補貨策略、啓發式演算法、積分法。

繆紹昌：修平技術學院軍訓室軍訓教官、東海大學統計系博士生

# 1. Introduction

Business environment and market-places have had some change in the past decades. Supply chain management (SCM) plays an important role in the business system. For increasing service level and reducing operating costs, most enterprises have contributed to the development of supply chain networks (SCNs) for frequent production-distribution replenishment (PDR) strategies. In this investigation, a multi-echelon supply chain network consists of manufacturers, warehouses, shopping centers and retailers for two deteriorating items. The phenomenon of deterioration of physical goods is very common in many inventory systems. It is well known that certain products such as medicines, volatile liquids, blood bank and foodstuff, etc., decrease under deterioration (vaporization, damage, spoilage, etc.) during their normal storage period. As a result, while determining the optimal inventory policy of those types of product, the loss due to deterioration cannot be neglected [20].

This paper investigates time-based models for SCNs that are appropriate when an organization is faced with time-dependent linear demand and want to maintain an inventory of the item most of the time. In these models, time and quantity can be considered interchangeable variables that determine the economic order interval (EOI) which indicates when item orders should be placed. Orders for items are placed based on time cycle which is decided by the multiplier $\lambda$ of the fixed delivery interval (FDI) $\tau$. The values of some variables must be chosen for an FDI system. Through all orders' calculation, the fixed review, produce/replenish period and the maximum inventory level can be developed. After a fixed period $\lambda\tau$ is determined, the stock of each item in inventory is also determined. Then an order is placed to replenish the stock and a production activity is to be carried out.

This paper emphasizes that SCM costs associated with distribution cycle stages potentially influence inventory level, operations, replenishment decisions, and management activities. The above costs are termed the "distribution cycle cost (DCC)". Accordingly, this paper emphasizes the role of production-distribution in SCN and hence examines whether retailers' replenishment in different distribution-frequency stages will significantly influence the SCN's performance. In this paper, the procurement-production strategy is to determine the ordering policy of raw material and the production strategies given by the demand that is a linear function of time, such as Goswami and Chaudhuri [13]. The major objective of this strategics is to minimize the average total cost by minimizing the inventory level via the optimal control of the distribution and the production, or the replenishment and the procurement. Heuristic algorithm and integration approach are developed to derive and model the multi-echelon SCN. In addition, the optimal policy is a derived efficiency to determine reorder period and replenishment amount while minimizing the average total cost and to decide when manufacturers should produce the products or stop, so that they know when to reduce working hours or salaries for workers due to the past consecutive months of rising unemployment in Taiwan. Here in Taiwan workers may be a group of people or machine associations in the factories. Finally, a detailed numerical case study about SCN distribution system for inventory control is presented to illustrate that the proposed model is effective and derive optimal solution. A sensitivity analyses is also performed in this study.

The remainder of this paper is thus organized as following: Section 2 reviews relevant literatures. Section 3 specifies assumptions and notations involved in our analytical model. Section 4 proposes both optimal problems of the production unit and the non-production unit. Section 5 presents a solution procedure to derive the equilibrium model for deteriorating items in a multi-echelon SCN. Then, a typical numerical example and sensitivity analysis are provided in Sections 6 and 7 respectively.

## 2. Literature review

### 2.1 Inventory models with deteriorating items

An EOI/ EOQ (Economic order quantity) model is developed for an item with a deterministic time-dependent demand pattern with a linear (positive) trend by [13]. They assumed that the inventory is replenished at a uniform rate. In recent years, some researches have studied two or multi-echelon inventory model with deteriorating items in supply chain. They derived an optimal joint total cost from an integrated perspective among the supplier and the producer or buyer. Huang and Yao [15] studied a deteriorating item in a supply chain system with a single vendor and multiple buyers and proposed a search algorithm to minimize the average total costs. Lin C. and Lin Y. [18] proffered a cooperative inventory with deteriorating items that considered and permitted completed back-order without the equal replenishments periods and present a procedure to find the optimal solution. Feng et al. [10] researched for a deteriorating inventory model for a single product under a two-echelon integrated supply chain with a manufacturer and a retailer. An algorithm for retailers to minimize the average total cost and for the manufacturer to optimize production time was derived.

Dong et al. [7] seek to investigate the optimal competitive pricing and replenishment policies in a two-echelon supply chain with one manufacturer and one retailer for a single deteriorating item. This research is motivated by the studies of Goswami and Chaudhuri [13], Yang and Wee [26], Christopher, M. [5], whose models are improved from four perspectives. Firstly, their integrated models are extended to a distributed scenario where the manufacturer and the retailer make their decisions independently and simultaneously. Secondly, the decision-makings of distribution and replenishment are integrated to balance a tradeoff. Thirdly, the fact that the deterioration in SCN inventory must be an indispensable part of the manufacturer's output is neglected in [26], which makes their works less precise. Fourthly, this paper have transformed their demand function into both of linear and constant time-elasticity demand functions respectively that suggested an optimal replenishment and distribution policy for the integrated supply chain network. Other researchers such as Dye et al. [8] discussed a two-warehouse inventory system for deteriorating items; Du et al. [9] focused on stock replenishment and shipment scheduling with deteriorating item for vendor management inventory (VMI) system.

## 2.2 Supply chain management in SCN

Supply chain management (SCM) problem has been discussed in many literatures. Since optimizing SCM in SCN as a whole is too complicated to solve efficiently in a single framework, it has been dealt with various forms in three sub-problems: dynamic manufacturing -production, procurement-production and inventory level for the SCN. A dynamic production and distribution problem and environmental performance addressed the simultaneous events of ordering sheet and inventory replenishment strategies in the environment of demand uncertainty ([2], [11], [17], [21]). For any firm, the first activity to begin with is to procure orders. A typical SCN is shown in Fig. 1.

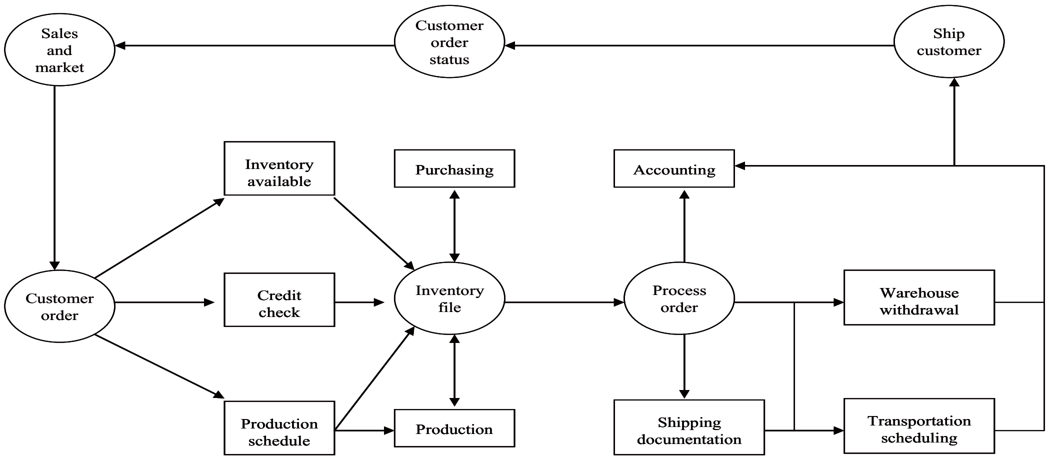


Fig. 1. Conceptual Model for a typical SCN (Source: Christopher [5]).

From Fig. 1, it is clear that the way of the orders generated and scheduled determines the performance of the production-distribution and replenishment strategies. Hence, the first step of SCM is to analyze the way the order-related activities to be carried out in SCN. To do this, the most important issues: such as the distribution-frequency method, replenish -ment cycle-time and the path of production start needing to be considered.

There is an important process in SCNs, which is also a production-distribution problem. Ganeshan [12] considered a two-level production/ distribution system that operates via many identical retails through the lower level. The model was a synthesis of three components: the inventory analysis at the retailers and the demand process at the warehouse and the inventory analysis at the warehouse. He developed a near-optimal inventory policy for a production/ distribution network with multiple suppliers replenishing a central warehouse which, in turn, distributes to a large number of retailers. Chen and Samroengraja [3] considered a distribution system with one-warehouse and $N$ retailers. Random demands occurred at the retailers only, and excess demands were completely backlogged. The retailers replenished their inventories from the warehouse, which in turn orders from an outside supplier that is assumed to have unlimited stock. They developed an approximate model to determine near-optimal control parameters for allocation policies. SCM techniques have been applied to solve these sub-problems ([6], [16], [19], [23], [24], [27]).

## 2.3 Effective SCM

Gunasekaran and Yusuf [14] have defined *Agility* in manufacturing as the capability of an organization to proactively establishing a virtual manufacturing with an efficient product development system to meet the changing market requirements, to maximize customer service level, and to minimize the cost of goods. It bears on the objective of having a competitive edge in a global market, as well as an increased chance of long-term survival and profit potential. Naylor et al. [22] and Youssef [28] have concurred about the need for manufacturers to be flexible and to cater to changing market conditions through agile manufacturing (AM) that must be supported by flexible people, processes and technologies. Effective SCM is an essential strategy for success in global markets and e-markets. The joint production and replenishment policies in an integrated environment have been analyzed in many literatures.

Different from the aforementioned literatures, this paper focuses on a joint distribution environment. The heuristic algorithm and integration approach are developed to derive and model the multi-echelon SCN. In addition, the optimal policy is derived efficiency to determine reorder period and replenishment amount while minimizing the average total cost and decide when manufacturers should produce the products or stop.

# 3. Assumptions and Problem descriptions

In view of the aforementioned researchers, it implies that all the supply chain members are integrated as a whole with identical objectives via full cooperation and information sharing. Abad and Jaggi [1] formulated a model of seller-buyer relationship and provided procedures for determining their policies under distributed system as well as integrated structure. This paper develops production-replenishment models for the multi-echelon SCN with various deteriorating items. The problems are considered in an integrated system and the production rate is constant. Our study will be extended from two perspectives as follows.

Firstly, the integrated inventory models in this paper are extended to a distributed case where interdependence between the manufacturer and the retailer is considered. Previous researches considering distributed SCN have taken notice of a leader-follower relationship between manufacturers and retailers, such as Chen et al. [4], Viswanathan and Piplani [25]. This paper considered a scenario where the manufacturer and the retailer make decisions independently and simultaneously to minimize their costs. The leader-follower relationship is relaxed to a symmetric relationship, i.e. neither player dominates the other. In this case, the manufacturer determines the distribution-frequency and production quantity, and simultaneously the retailer specifies the order-cycle and replenishment number that refers to the number of replenishment decisions that the retailer makes during the entire cycle. It also implies that how many delivery decisions should be made by the manufacturer and warehouse.

Secondly, in the context of multiple retailers, both distribution period and relative cost return to schemes that may have a revision. This paper considers a manufacturer selling to multiple retailers, each with a distinct demand and a positive cost. Suppose, for simplicity, that the manufacturer has monopoly power over the SCN system. The manufacturer offers a set of distribution period to the retailers as a take-it-or-leave-it offer. The manufacturer's cost is minimized if total system cost is minimized and each retailer receives only manufacturer's distribution. This is possible because the various distribution period contracts depends on the retailer's demand distribution. Hence, when distribution period is allowed, coordination and win-win can be achieved by a contract signed by the manufacturer and the retailer (or warehouse). Transportation times are deterministic and this paper assumes that the retailers face stationary distribution and independent random demand. Manufacturer limits himself to one-for-one replenishment policies. If demands occur while the distribution center has already finished distribution, shipments to retailers (or warehouses) are delayed. They are served according to a first-come, first-served policy.

Finally, the factors of distribution-frequency and replenishment are taken into account simultaneously in the model. The integration approach provides an optimal solution for a real supply chain system which is decentralized from most of supply chain systems. Empirical observations in the market place show that the demand of products affected the retailer’s lot size, whereas the lot size is dependent on the distribution-frequency of products. Therefore, there is interdependence between the delivery on terms and replenishing decisions. This investigation considers a multi-echelon supply chain model consisting of manufacturer with a distribution center (supplier), multiple retailers and various products with deteriorating merchandise. This paper proceeds on the following assumptions which are frequently used in the aforementioned literatures:

- The members' demands are independent in the horizontal level. Neither dominates the other. To include multiple manufacturers is easy to be generalized.
- Shortage is not allowed and there is a limit in distribution-frequency.
- The replenishment for the SCN is limited to the frequency of distribution.
- The production is finite and greater than the demand rate.
- Expected deterioration rates are different between the production unit and the non-production unit. The rates are related to time and inventory.
- Demand rate is an increasing linear function of time for all members.

The main parameters and notations involved in the analytical model are specified as follows:

| | |
|---|---|
| $n$ | Member amount of supplier chain; |
| $m$ | Amount of the type for the merchandizes in SCN; |
| $e$ | Amount of the echelon in SCN; |
| $i$ | Number of supplier chain members, $i$=0 that is the production unites and $i$=1,2······,$n$ that are the non-production units; |
| $j$ | Number of the type for the merchandizes in SCN, $j = 1,2\cdots\cdots,m$; |
| $k$ | Number of the echelon in SCN, $k = 1,2\cdots\cdots,e$; |
| $h_{j,i,k}$ | Holding cost of per unit time per item $j$ for the member $i$ at echelon k; |
| $p_{j,i,k}$ | Deterioration cost of per unit time per item j for the member i at echelon k; |
| $\theta_{j,i,k}$ | Deterioration rate of per unit time per item j for the member i at echelon k; |
| $CP_{j,i,k}$ | Manufacturing cost of per unit time per item j for the member i at echelon k; |
| $CO_{j,i,k}$ | Distribution cost of per item j for the member i at echelon k by the supplier (manufacturer or warehouse); |
| $Cb_{j,i,k}$ | Set up cost of per item j for the member i at echelon k by the supplier or one's own production; |
| $K_{j,i,k}$ | It is defined both the production rate and processing/packaging rate of per unit time per item j for the member i at echelon k; |
| $DN$ | It represents a maximum delivering number at maximum postponement delivering time $DT$; |
| $T$ | The total length of observation cycle time; |

| | |
|---|---|
| $t_{1j,i,k}$ | It is defined the length of production time and the length of processing and packaging time for per item j by the member i at echelon k; |
| $t_{2j,i,k}$ | It is defined the length of non-production time for per item j by the member i at echelon k; |
| $f_{j,i,k}(t)$ | Demand rate is an increasing linear function of per item j for the member i at echelon k by time t, such as $a+bt$ with $a \geq 0$, $b > 0$; |
| $Q_{j,i,k}(t)$ | Inventory level of per item j for the member i at echelon k by time $t \in [0, t2_{j,i,k}]$; |
| $C_{j,i,k}(t)$ | Average total cost per unit time of per item j for the member i at echelon k by time t; |
| $DSC(DT,D)$ | Total cost of SCN system by delivering number $D$ at maximum postponement delivering time $DT$, $D=1,2...DN$. |

# 4. The optimization model for supply chain network

This study considers a distributing system of one manufacturer, multiple retailers, and various products with deteriorating merchandise. A typical multi-echelon supply chain network and the relevant decision process are shown in Figure 2, in which the solid/dotted line arrow means the product/distribution or demand/information flow, the upward/ downward round shape means the production or consumption process, and first level represent the warehouses at shopping centers and retailers respectively. The process can be depicted as follows:

- The manufacturer and process-assembling warehouse determine the lengths of production/processing time and the lengths of time for packaging products and stopping producing.
- Simultaneously, the retailer and shopping center demands occur.
- The manufacturer and process-assembling warehouse determine inventory level, producing and replenishment strategy in the SCN.
- Replenishment arrives at warehouses of shopping centers and retailers. Shipments are released. Consumer demand occurs.
- Fixed and variable costs are charged.

## 4.1 Forecasting and Modeling methods

### 4.1.1 Forecasting

Replenishment policy was concerned with finding optimal inventory policies. These policies are, in part, dependent upon some forecast of sales or use of the items of interest. Forecasting is an essential component of any successful enterprise. However, forecasting need not be associated solely with problems of replenishment control. Other areas where forecasting plays an important role include marketing, financial planning, and production planning. Indeed, managerial decisions seldom are modes in the absence of some form of forecasting. Thus a forecast is a basic tool to aid managerial decision making.

The paper supposes that the generating process of the observed time series can be represented by a linear trend superimposed with random fluctuations. Denote the slope of the linear trend by *b*, where the slope is called the trend factor. The model is represented by

$$f_{j,i,k}(T) = a_{j,i,k} + b_{j,i,k} \cdot T + eT$$

where *f(T)* is the random demand that is observed at time *T*, *a* is a constant, *b* is the trend factor, and *eT* is the random error occurring at time *T* (assumed to have expected value equal to zero and constant variance).

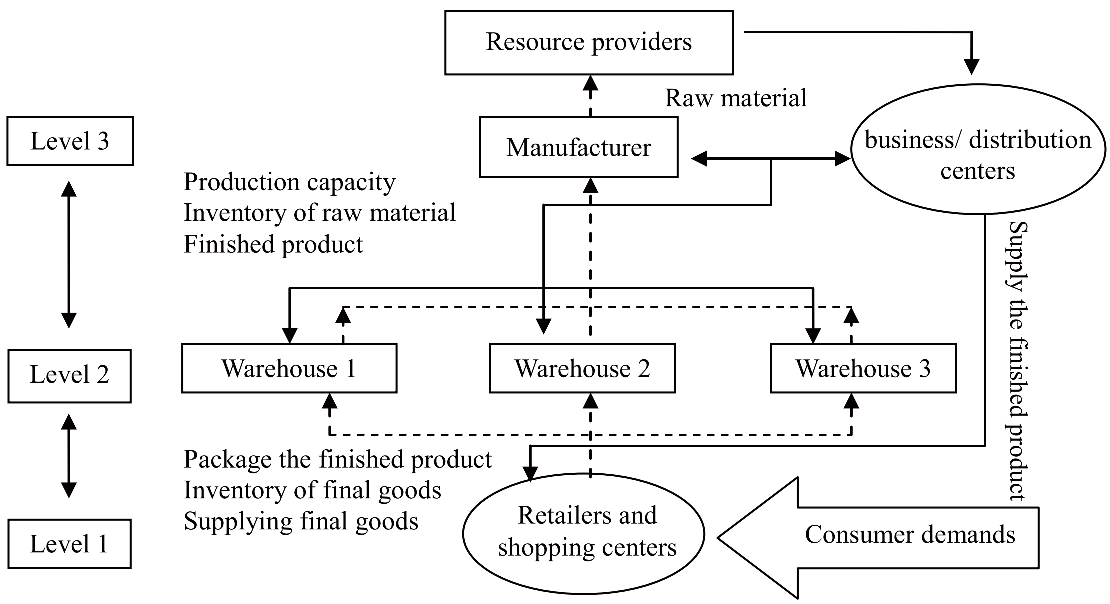


Figure 2. Conceptual Model for Supply Chain Networks

## Regression Analysis

The SCN manager has noted that there is a rather interesting linear relationship between the number of cycle time of demand orders and the number sold through distribution during the past months. Regression analysis establishes a temporal relationship for the forecast variables. The variable to be predicated (demand $f(T)$ ) is referred to as the dependent variable, while the variable used in predicting (time $T$ or price $P$ ) is called the independent variable. The usual method for identifying the "best" fitted line is the Method of Least Squares (MLS). The parameters $a$ and $b$ are estimated from the following formulas:

$$b_{j,i,k} = s(\Sigma_{t=1}^{s} f_{j,i,k}(T_t)\cdot T_t) / (s\Sigma_{t=1}^{s} T_t^2 - (\Sigma_{t=1}^{s} T_t)^2) = slope$$

and

$$a_{j,i,k} = (\Sigma_{t=1}^{s} f_{j,i,k}(T_t)/s) - b_{j,i,k}\cdot(\Sigma_{t=1}^{s} T_t/s) \quad (1)$$

, where $s$ is the number of periods of demand data included in the calculation. The degree of linear association of the forecast variable to the time variable is determined by the correlation coefficient r (positive), one variable tends to increase as the other increases, such as [27]. When the coefficient is negative, one variable tends to decrease as the other increases, such as Dong et al. [7]. The following formula is used to compute both the correlation coefficient r and the standard deviation SD:

$$r_{j,i,k}^2 = [\Sigma_{t=1}^{s}[(f_{j,i,k}(T_t) - \Sigma_{t=1}^{s}(f_{j,i,k}(T_t)/s))\cdot (T_t - (\Sigma_{t=1}^{s} T_t/s))]]^2 / (\Sigma_{t=1}^{s}[f_{j,i,k}(T_t) - \Sigma_{t=1}^{s}(f_{j,i,k}(T_t)/s)])^2 (\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)])^2$$

and

$$SD_{j,i,k}^2 = [1/(s-2)]\cdot\{\Sigma_{t=1}^{s}(f_{j,i,k}(T_t) - [f_{j,i,k}(T_t)/s)]) - [[\Sigma_{t=1}^{s}[(f_{j,i,k}(T_t) - \Sigma_{t=1}^{s}(f_{j,i,k}(T_t)/s))\times(T_t - (\Sigma_{t=1}^{s} T_t/s))]]^2 / (\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)])^2]\} \quad (2)$$

Once the linear regression line and the standard deviation are determined, control limits can be established by confidence interval for the parameters $a$ and $b$ of the t distribution with (s-2) degrees of freedom from the variable $a$ and b in this paper. This paper shall find two (1-α)% confidence intervals for $a$ and $b$:

$$b_{j,i,k} - \left(t_{\alpha/2}\cdot SD_{j,i,k} \Big/ \sqrt{\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)]^2}\right) = b_{j,i,k}^L < b < b_{j,i,k}^U = b_{j,i,k} + \left(t_{\alpha/2}\cdot SD_{j,i,k} \Big/ \sqrt{\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)]^2}\right)$$

and

$$a_{j,i,k} - \left(t_{\alpha/2}\cdot SD_{j,i,k}\cdot\sqrt{\Sigma_{t=1}^{s} T_t^2} \Big/ \sqrt{s\cdot\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)]^2}\right) = a_{j,i,k}^L < a < a_{j,i,k}^U = a_{j,i,k} + \left(t_{\alpha/2}\cdot SD_{j,i,k}\cdot\sqrt{\Sigma_{t=1}^{s} T_t^2} \Big/ \sqrt{s\cdot\Sigma_{t=1}^{s}[T_t - (\Sigma_{t=1}^{s} T_t/s)]^2}\right) \quad (3)$$

Hence, this paper shall find the center control limit can be computed using the formula:

$$f_{j,i,k}(t) = a_{j,i,k} + b_{j,i,k}\times t \quad (4)$$

## 4.2 The optimization model of the production units

The average total cost function came from the concept of $dC_{j,i,k}(t)/dt_{2,j,i,k}=0$ and can be used for finding the SCN average total cost function that has sufficient and necessary conditions. It is ensured that the SCN average total cost function will have positive solutions; moreover, obtains an interior local minimum from $dC_{j,i,k}(t)/dt_{2,j,i,k}=0$. Finally, this paper finds the minimum solution for the SCN average total cost per unit time in this paper. This paper changes some notations of the deterministic order-level model with finite rate of replenishment without backlogging is developed with the following notations in [13]. The time-varying inventory levels for the production unites were given by:

$$dQ_1/dt+\theta\cdot Q_1=K-(a+bt),\quad for\ \ 0\le t\le t_1\ ;\ Q_1(0)=0, \tag{5}$$

$$dQ_2/dt+\theta\cdot Q_2=-(a+bt),\quad for\ \ t_1\le t\le t_2\ ;\ Q_2(t_2)=0, \tag{6}$$

Eq. 5 represents the time-varying inventory level for the production units. After calculation, this paper obtain inventory levels

$$Q_1(t)=[(K-a)\cdot\theta(e^{\theta\cdot t}-1)-b\cdot\theta\cdot t\cdot e^{\theta\cdot t}+b(e^{\theta\cdot t}-1)]/\theta^2 e^{\theta\cdot t}. \tag{7}$$

Eq. 6 represents the time-varying inventory level for the production units. After calculation, this paper obtain inventory levels

$$Q_2(t)=[a\cdot\theta(e^{\theta\cdot t_2}-e^{\theta\cdot t})+b\cdot\theta(t_2\cdot e^{\theta\cdot t_2}-t\cdot e^{\theta\cdot t})-b(e^{\theta\cdot t_2}-e^{\theta\cdot t})]/\theta^2 e^{\theta\cdot t}\ . \tag{8}$$

There is a relation between $Q_1$ and $Q_2$ by the following formula:

$$Q_1(t_1)=Q_2(t=0). \tag{9}$$

According to the above assumptions and notions, Figure 3 gives the SCN unites' time-varying inventory level in a cycle. The inventory level decreases when $t > t_1$ in a replenishment period. It can be given by Eqs. 5~9. This paper use the following additional notation throughout the paper. This paper establishes the models when it considers multi-echelon units and various products cases. So adds the subscript j, i and k. Average total cost per unit time for the SCN units is derived as following:

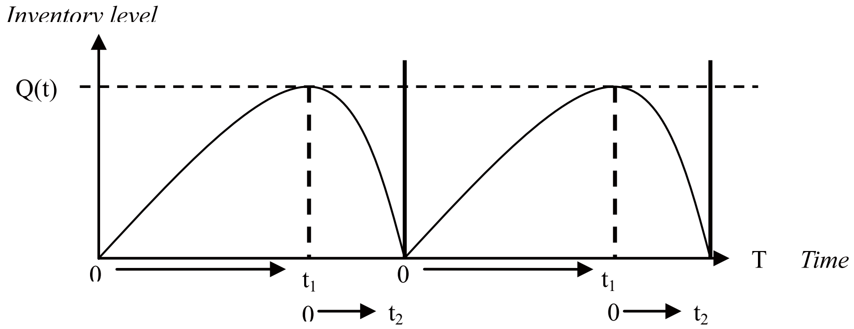


Figure 3 Time-varying inventory curve for the SCN unites

Ordering cost + Distribution cost =

$$(CO_{j,i,k} + Cb_{j,i,k})/t_{2,j,i,k}$$

Purchasing cost + manufacturing cost =

$$K_{j,i,k} \cdot CP_{j,i,k} + Cb_{j,i,k}/t_{1,j,i,k}$$

Holding cost =

$$h_{j,i,k} \cdot [(\int_0^{t_{2\,j,i,k}} Q_2(t)dt / t_{2\,j,i,k}) + (\int_0^{t_{1\,j,i,k}} Q_1(t)dt / t_{1\,j,i,k})]$$

Deterioration cost =

$$p_{j,i,k} \cdot \theta[(\int_0^{t_{2\,j,i,k}} Q_2(t)dt / t_{2\,j,i,k}) + (\int_0^{t_{1\,j,i,k}} Q_1(t)dt / t_{1\,j,i,k})]$$

Average total cost

$$C_{j,i,k}(t_1(t_2)) = (h_{j,i,k} + p_{j,i,k}) \times [(\int_0^{t_{2\,j,i,k}} Q_2(t)dt/t_{2\,j,i,k} + \int_0^{t_{1\,j,i,k}} Q_1(t)dt)/t_{1\,j,i,k}] + K_{j,i,k} \cdot CP_{j,i,k} + Cb_{j,i,k}/t_{1\,j,i,k} + (Cb_{j,i,k} + CO_{j,i,k})/t_{2\,j,i,k} \quad (10)$$

## 4.3 The optimization model of the non-production units

Average total cost per unit time for the non-production units by

$$C_{j,i,k}(t2_{j,i,k}) = [(h_{j,i,k} + p_{j,i,k}\theta) \int_0^{t2_{j,i,k}} Q_{j,i,k}(t)dt + Cb_{j,i,k} + CO_{j,i,k}]]/t2_{j,i,k} \quad (11)$$

## 4.4 The optimization model of the SCN system

Our goal is to find the minimum solution for the SCN average total cost in the production system. Thus, this paper attempts to finish SCN average total cost function per unit time:

$$ATC(DT, D) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{e} C_{j,i,k}\left(\text{ceiling}\left[t_{1\,j,i,k}(t_{2\,j,i,k})/(DT/D)\right]\right) \cdot DT/D \quad (12)$$

The objective function was transferred to the SCN system average total cost function as Eq. 12.

$$DSC(T,D) = \sum_{j=1}^{m} \sum_{i=1}^{n} \sum_{k=1}^{e} T \cdot ATC(DT,D) / (\text{ceiling}\left[(t_{2\,j,i,k} + t_{1\,j,i,k})/(DT/D)\right] \cdot DT/D) \quad (13)$$

# 5. Solution procedure

Two echelon SCN models with distribution and producing of their goods are considered for deteriorating items under linear demand related to time. The optimal solution procedure for the above- described models includes the following steps in subsections 5.1 and 5.2:

## 5.1 Solution procedure for Goswami and Chaudhuri's model

The inventory is replenished at a finite uniform rate *K*, which exceeds the demand rate at any instant during the production period [0, *t*]. That means $K \geq a + bt$, $t \in [0, t_1]$. This paper adopt a weak condition, suppose $Q(t) \geq 0$ for $t \in [0, t_1]$. In both conditions, production is stopped at $t_1$ and the excess stock accumulated in $[0, t_1]$ is used to satisfy demands in $[t_1, t_2]$. The initial inventory is zero. Production starts at $t = 0$ and continues until $t = t_{1,j,i,k}$.

Then, the differential equations governing both systems are as Eqs. 5 and 6,

$$\frac{dQ}{dt} = K - (a + bt), for\ 0<t< t_1,\ Q(0)=0, \text{ and}$$

$$\frac{dQ}{dt} = -(a + bt), for\ t_1<t< t_2,\ Q(t_2)=0, \quad (14)$$

therefore,

$$Q(t) = Kt - at - bt^2/2, \quad \text{for } 0\leq t\leq t_1, \text{ and}$$

$$Q(t) = Kt_1 - at - (bt^2/2), \quad \text{for } t_1< t< t_2. \quad (15)$$

There is a relation between $t_1$ and $t_2$ by the following formula:

$$K \cdot t_1 = a \cdot t_2 + b \cdot t_2^2/2 \ , \quad (16)$$

where $t_1$ and $t_2$ represent time points. From the above formula, strategies of the each level in SCM system can be evaluated by the following steps.

**Step 1:**

Eq. 14 is used to simplify the result of Eq. 10, with the following abbreviations

$$\mathrm{U} = \frac{-b^2}{8K}, \mathrm{V} = \frac{-ab}{2K} + \frac{b}{3}, \mathrm{W} = \frac{a}{2} - \frac{a^2}{2K},$$

then average total cost function becomes

$$C(t_1(t_2)) = (h + p \cdot \theta)\left(\mathrm{U} \cdot t_2^3 + \mathrm{V} \cdot t_2^2 + \mathrm{W} \cdot t_2\right)$$
$$+ (K \cdot t_1 \cdot CP + Cb + CO)/t_2 \quad (17)$$

**Step 2:**

From $K \geq a + bt$ for $t \in [0, t_1]$, this paper get $(K-a)/b \geq t_1$, by (7), this paper have

$$\mathrm{y}_1 = (\sqrt{K^2 + (K-a)^2} - a)/b \geq t_2.$$

Our purpose is to determine the optimal values of $t_1$ and $t_2$ so that they can minimize the average total cost function $C(t_2)$, with $t_2 \in (0, \mathrm{y}_1]$. In the Eq. 14, since $Q(t) \geq 0$ on $[t_1, t_2]$, therefore $(K- a)/b \geq t_2 - t_1$ and $\mathrm{y}_2 = (K-a)/b \geq t_2$. Our goal is to determine the optimal values of $t_1$ and $t_2$ so that they can minimize the average total cost function $C(t_2)$, with $t_2 \in (0, \mathrm{y}_2]$.

At the beginning our purpose are to find the sufficient and necessary conditions to ensure $dC/dt2 = 0$ have positive solutions.

Since

$$\frac{dC(t_2)}{d\,t_2} = (h + p \cdot \theta)\left(3\mathrm{U} \cdot t_2^2 + 2\mathrm{V} \cdot t_2 + \mathrm{W}\right) - (K \cdot t_1 \cdot CP + Cb + CO)t_2^2$$

to simplify the subscript and study the properties of d$C$ /d$t2$, let

$$\mathrm{g}(\mathrm{z}) = \left(3\mathrm{U} \cdot \mathrm{z}^4 + 2\mathrm{V} \cdot \mathrm{z}^3 + \mathrm{W}\mathrm{z}^2\right) - (K \cdot t1 \cdot CP + Cb + CO)(h + p \cdot \theta),$$

for z∈(-∞, ∞), this paper get,

$$g'(\mathrm{z}) = 12\mathrm{U}\mathrm{z}^3 + 6\mathrm{V}\mathrm{z}^2 + 2\mathrm{W}\mathrm{z} \cdot$$

Now to solve g′(z)=0, the paper has the solutions {[$(9V^2\text{-}24UW)^{1/2}$+3V]/(-12U)] , 0, [$(9V^2\text{-}24UW)^{1/2}$+3V]/(-12U)]}.

From g(0)= (K·$t1$·$CP$+ $Cb$+ $CO$) / (-($h$+ $p$·$\theta$)) , to know that the sufficient and necessary conditions to ensure g(z)=0 at least has a positive solution $t_2$.

**Step 3:**

Put boundary value and $t_2$ into Eq. 17, the minimum average total cost will be an optimum solution. The method as described above can avoid making Goswami and Chaudhuri's mistake. Thus obtained

$$\mathrm{Min}\left\{ \begin{array}{l} C\left(\left(\sqrt{K^2 + (K-a)^2} - a\right)/b\right), \\ C(2(K-a)/b), C(t_2) \end{array} \right\} . \quad (18)$$

[13] observed some numerical results that are shown in Table 1.

Table 1 Goswami and Chaudhuri's numerical results

| | $t1^*$ | $t2^*$ | $C^*$ | $t1^*/t2^*$ |
|---|---|---|---|---|
| b=0 | 0.297 | 1.98 | 10.1 | 15 % |
| b=4 | 0.404 | 1.395 | 12.601 | 29 % |

Table 2 Numerical results for $t2$ and $C(t2)$

| a | b | K | $C_1$ | $C_3$ | G & G's $C(t2)$ | C $t2$) | $t2$ | $y_1$ | $C(y_1)$ | $y_2$ | $C(y_1)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 4 | 40 | 1 | 10 | 12.7 | 11.113 | 1.373 | 11.624 | 98.826 | 17 | 107.486 |

Using the data set given in [13], this paper compares results from its own algorithm with those from [13]. Some numerical results are shown in Table 2. Numerical results for $t_2$ and $C(t_2)$ have given proof of their mistake.

## 5.2 Solution procedure for production-distribution model

From the above-described models, the following steps can evaluate strategies of the each level in SCN system. Nevertheless, the inventory is replenished at a finite uniform rate $K$, which exceeds the demand rate and deterioration amount at any instant during the production period [0, $t$]. This paper adopted a weak condition, suppose $Q(t)$ ≥0 for $t$ ∈[0, $t_1$]. In both conditions, this paper obtained the sufficient and necessary conditions, $K \geq a + [\, b\, t\, e^{\theta t} \div (e^{\theta t}\text{-}1)\,]\text{-}\,(b/\theta),\ t > 0$.

**Step 1:**

At the beginning our purpose are to find the sufficient and necessary conditions to ensure d$C$/d$t2$ =0 have positive solutions.

Since

$$C(t_2)=\left[(h+p\cdot\theta)/t_2\theta^2\right]\left(a(e^{\theta\cdot t_2}-1)+{}^{b}\!/_{\theta}\right.$$
$$\left.(1-e^{\theta\cdot t_2})+b\cdot t_2(e^{\theta\cdot t_2}-(\theta\cdot t_2/2))-a\cdot\theta\cdot t_2\right), \quad (19)$$

then

$$\frac{dC(t_2)}{d\,t_2}=\left((h+p)/\theta^2\right)\left[a\theta\cdot t_2e^{\theta\cdot t_2}-ae^{\theta\cdot t_2}+a-\right.$$
$$\left.(b/\theta)(1-e^{\theta\cdot t_2})+b\cdot t_2e^{\theta\cdot t_2}(\theta-1)-(b\cdot t_2\cdot\theta/2)\right]$$
$$-(Cb+CO)=0\ . \quad (20)$$

**Step 2:**
By Eq. 9, this paper gets

$$Q_1(t_1)-Q_2(t=0)=0\,.$$

Therefore, this paper can determine the optimal values of $t_1$ and $t_2$ so that they can minimize the average total cost function $C_{j,i,k}$ $(t_1(t_2))=C_{j,i,k}$ $(t_2)$ as Eq. 20. $t_1$ and $t_2$ represent time lengths.

**Step 3:**

Put boundary value and $t_2$ into Eq. 19, the minimum average total cost will be an optimum solution as Eq. 21.

$$\text{Min}\left\{\begin{matrix}C\left((\sqrt{K^2+(K-a(1-\theta))^2}-a(1-\theta))/b(1-\theta)\right),\\ C(2(K-a(1-\theta))/b(1-\theta)),C(t_2)\end{matrix}\right\} \quad (21)$$

**Step 4:**
Calculate the demand of the center of production-distribution by

$$f_o(t)=\sum_{i=1}^{n-1}Q_i(t).$$

In addition, let $D$ =1 and r=0 (recursive number). Judge whether it can finish the solution of VMI-SCM system optimization or not by $DSC_r > DSC_{r-1}$, if yes, then the calculation is terminated, otherwise let $D$=$D$+1 and return to step 1.

# 6. Numerical Illustration

This research seeks to investigate the optimal PDR strategies of some fruits and vegetables in Taiwan as research object. The paper distinguishes SCN into an upper-stream production/distribution center and three downstream business/ distribution stations. The census of every level demand is shown in Table 3.

Table 3 The statistics of demand and time-length (10 kilograms/daily)

| Time-length | Retailer 1 demand | Retailer 2 demand | Retailer 3 demand | Center demand |
|---|---|---|---|---|
| 1 | 10 | 11 | 3 | 24 |
| 2 | 14 | 14 | 6 | 34 |
| 4 | 22 | 22 | 10 | 54 |
| 3 | 18 | 18 | 8 | 44 |
| 1 | 10 | 9 | 5 | 24 |
| 1 | 9 | 10 | 3 | 22 |
| 1 | 11 | 10 | 4 | 25 |
| 3 | 20 | 20 | 9 | 49 |
| 4 | 21 | 21 | 10 | 52 |
| 2 | 13 | 13 | 6 | 32 |

From Table 3 made calculations by Eqs. 1~4 that can fine the parameters of every level are shown in Table 4.

Table 4 The parameters of the center and stations

| Organization | Center *0* | Station *1* | Station *2* | Station *3* |
|---|---|---|---|---|
| $a_i$ | 14 | 6 | 6 | 2 |
| $b_i$ | 10 | 4 | 4 | 2 |

In this research, there is a production/distribution center and three business/distribution stations system to illustrate that our proposed model is feasible and efficient. The unit cost is showed in Table 5.

Table 5 Related unit cost for production-distribution center and three retailers system

| Parameters | Station *1* | Station *2* | Station *3* | Center *0* |
|---|---|---|---|---|
| $K_{j,i,k}$ | 40 | 40 | 50 | 150 |
| $T$ | 30 | 30 | 30 | 30 |
| $Cb_{j,i,k}$ | 4 | 4 | 3 | 1 |
| $CO_{j,i,k}$ | 0.05 | 0.05 | 0.2 | 0.024 |
| $CP_{j,i,k}$ | 0.1 | 0.1 | 0.1 | 0.05 |
| $\theta_{j,i,k}$ | 0.00428 | 0.001 | 0.001 | 0.002 |
| $p_{j,i,k}$ | 20 | 20 | 10 | 5 |
| $h_{j,i,k}$ | 0.011 | 0.0392 | 0.02575 | 0.0747 |

The production/distribution center (or business/distribution stations) packs products, and adopted the delivering frequency that is twice daily (at 5 o'clock in the morning and in the afternoon). To satisfy the ordering demand of the downstream business/distribution stations and retailers, the ordering demand happens before the delivering echelon in current period, it would be delivered in the current period. Through mathematical calculations from Eq. 7 to Eq. 13, the results are shown in Table 6 and 7.

Table 6 The solution of the boundary value and strategies of every level in the SCN (10,000 New Taiwan dollars /10 kilograms /day)

| Organization | $a_i$ | $b_i$ | $y_1$ | $y_2$ | $t2$ | $C(y_1)$ | $C(y_2)$ | $C(t2)$ |
|---|---|---|---|---|---|---|---|---|
| Station *1* | 6 | 4 | 11.624 | 17 | 1 | 86.281 | 94.107 | 8.287 |
| Station *2* | 6 | 4 | 11.624 | 17 | 3 | 11.107 | 13.542 | 4.109 |
| Station *3* | 2 | 2 | 33.66 | 48 | 4 | 20.049 | 23.766 | 2.287 |
| Center *0* | 14 | 10 | 18.847 | 27.2 | 1 | 59.704 | 67.134 | 3.019 |

Table 7 The optimum solutions of every level in the SCN (10,000 New Taiwan dollars /10 kilograms /day)

| Organization | $t2^*$ | $C(t2^*)$ | $t1$ | $Q_i\,(t1)$ | $DSC$ |
|---|---|---|---|---|---|
| Station *1* | 1 | 8.287 | 0.2 | 6.723 | 17.702 |
| Station *2* | 3 | 4.109 | 0.9 | 28.983 | |
| Station *3* | 4 | 2.287 | 0.48 | 21.392 | |
| Center *0* | 1 | 3.019 | 0.127 | 17.154 | |

This paper explains the result in Table 7 as follows:

- The optimum strategics of the production/distribution center in the upper stratum is to determine production cycle, the daily twice- delivering frequency and maximal stock level is maintained at 171.54 kilograms. The average unit cost is 3.0190 (unit is 10,000 New Taiwan dollars) within each production cycle on the periodic review time 30 days.
- The center adopts the daily once-delivering frequency for business/ distribution station *1* whose maximal stock level is maintained at 67.23 kilograms. The average unit cost is 8.287 (unit is 10,000 New Taiwan dollars).
- The center adopts the every three days

once-delivering frequency for business/ distribution station *2* whose maximal stock level is maintained at 289.83 kilograms. The average unit cost is 4.109 (unit is 10,000 New Taiwan dollars).

- The center adopts the every four days once-delivering frequency for business/distribution station *3* whose maximal stock level is maintained at 213.92 kilograms. The average unit cost is 2.287 (unit is 10,000 New Taiwan dollars).

# 7. Sensitivity analysis

This section probes into relative cost parameters whose change causes influence such as the production/ distribution period (*t1* and *t2*), producing/ delivering frequency, maximal stock level and system average unit cost, etc. in the system. In addition, the demand parameters ($a_i$ and $b_i$) have already been probed in the section 2. The influence caused is illustrated in Table 8 and 9 and explain as follows:

Table 8 The sensitivity analysis of the solution of every level in the SCN (10,000 New Taiwan dollars /10 kilograms /day)

| Organization | Change of the parameter | $t2^*$ | $C(t2^*)$ | $t1$ | $Q_i(t1)$ | $t1^*/t2^*$ |
|---|---|---|---|---|---|---|
| Station *1* | $h_i$×10 | 0.987↓ | 18.16↑ | 0.197↓ | 6.612↓ | 0.19 ↓ |
| Station *2* | | 1.244↓ | 14.214↑ | 0.264↓ | 14.214↓ | 0.2 ↓ |
| Station *3* | | 2.277↓ | 7.169↑ | 0.195↓ | 7.196↓ | 0.1 ↓ |
| Center *0* | | 0.392↓ | 5.795↑ | 0.042↓ | 5.663↓ | 0.13 ↓ |
| Station *1* | $h_i$×0.1 | 1.005↑ | 8.252↓ | 0.201↑ | 6.764↑ | 0.2 — |
| Station *2* | | 4.359↑ | 3.93↓ | 1.604↑ | 49.395↑ | 0.37 ↑ |
| Station *3* | | 5.871↑ | 2.244↓ | 0.924↑ | 40.423↑ | 0.16 ↑ |
| Center *0* | | 1.884↑ | 2.571↓ | 0.294↑ | 39.576↑ | 0.156 ↑ |
| Station *1* | $P_i$×10 | 0.368↓ | 21.605↑ | 0.062↓ | 2.099↓ | 0.17 ↓ |
| Station *2* | | 1.703↓ | 5.438↑ | 0.401↓ | 13.299↓ | 0.24 ↓ |
| Station *3* | | 2.515↓ | 2.789↑ | 0.227↓ | 10.395↓ | 0.09 ↓ |
| Center *0* | | 0.74↓ | 3.575↑ | 0.087↓ | 11.838↓ | 0.12 ↓ |
| Station *1* | $P_i$×0.1 | 2.46↑ | 4.416↓ | 0.672↑ | 21.935↑ | 0.27 ↑ |
| Station *2* | | 3.479↑ | 3.978↓ | 1.127↑ | 35.771↑ | 0.34 ↑ |
| Station *3* | | 4.449↑ | 2.244↓ | 0.574↑ | 25.173↑ | 0.13 ↑ |
| Center *0* | | 1.048↑ | 2.955↓ | 0.134↑ | 18.18↑ | 0.128 ↑ |
| Station *1* | $CP_i$×10 | 3.706↑ | 21.083↑ | 1.243↑ | 39.161↑ | 0.34 ↑ |
| Station *2* | | 5.693↑ | 13.901↑ | 2.475↑ | 71.894↑ | 0.44 ↑ |
| Station *3* | | 9.396↑ | 9.315↑ | 2.142↑ | 105.889↑ | 0.23 ↑ |
| Center *0* | | 1.981↑ | 42.387↑ | 0.316↑ | 42.429↑ | 0.16 ↑ |
| Station *1* | $CP_i$×0.1 | 1.67↓ | 6.671↓ | 0.39↓ | 12.955↓ | 0.23 ↓ |
| Station *2* | | 2.506↓ | 4.369↓ | 0.69↓ | 22.509↓ | 0.28 ↓ |
| Station *3* | | 4.019↓ | 2.129↓ | 0.484↓ | 22.999↓ | 0.12 ↓ |
| Center *0* | | 0.778↓ | 11.229↓ | 0.093↓ | 12.567↓ | 0.12 ↓ |

- The change of the parameters influence on the average unit cost: The change of the average unit cost $C(t2^*)$ depends on that of all parameters. They have the same change direction. As distribution cost $CO_i$ gets larger (smaller), the average unit cost becomes larger (smaller).
- The change of the parameters influence on the periodic review time $t2^*$: The change of the periodic review time $t2^*$ depends on that of parameters $CO_i$, $Cb_i$, $CP_i$ and $K_i$. They have the same change direction. As purchasing cost $CP_i$ gets larger (smaller), the periodic review time becomes larger (smaller). In addition, as the production rate (or processing/packaging rate) gets larger (smaller), though the periodic production time $t1$ gets smaller (larger), but the maximal stock level effect is more (loss) intense when the periodic review time gets larger (smaller).
- The change of the periodic review time $t2^*$ depends on that of parameters $h_i$ and $P_i$. But they have opposite change directions. As deterioration cost $P_i$ gets larger (smaller), the periodic review time becomes smaller (larger).

Table 9 The sensitivity analysis of the solution of every level in the SCN (10,000 New Taiwan dollars /10 kilograms /day)

| Organization | Change of the parameter | $t2^*$ | $C(t2^*)$ | $t1$ | $Q_i(t1)$ | $t1^*/t2^*$ |
|---|---|---|---|---|---|---|
| Station *1* | $Cb_i \times 10$ | 3.706↑ | 21.083↑ | 1.243↑ | 39.161↑ | 0.34 ↑ |
| Station *2* | | 5.693↑ | 13.901↑ | 2.475↑ | 71.894↑ | 0.44 ↑ |
| Station *3* | | 8.096↑ | 7.259↑ | 1.635↑ | 80.548↑ | 0.2 ↑ |
| Center *0* | | 1.731↑ | 34.635↑ | 0.261↑ | 35.202↑ | 0.15 ↑ |
| Station *1* | $Cb_i \times 0.1$ | 1.67↓ | 6.671↓ | 0.39↓ | 12.955↓ | 0.24 ↓ |
| Station *2* | | 2.506↓ | 4.369↓ | 0.69↓ | 22.509↓ | 0.28 ↓ |
| Station *3* | | 4.457↓ | 2.616↓ | 0.576↓ | 27.586↓ | 0.13 ↓ |
| Center *0* | | 0.866↓ | 14.345↓ | 0.106↓ | 14.345↓ | 0.12 ↓ |
| Station *1* | $CO_i \times 10$ | 3.028↑ | 15.744↑ | 0.912↑ | 29.359↑ | 0.3 ↑ |
| Station *2* | | 4.585↑ | 10.399↑ | 1.739↑ | 53.078↑ | 0.38 ↑ |
| Station *3* | | 7.299↑ | 6.09↑ | 1.385↑ | 66.685↑ | 0.19 ↑ |
| Center *0* | | 1.451↑ | 27.741↑ | 0.206↑ | 27.741↑ | 0.14 ↑ |
| Station *1* | $CO_i \times 0.1$ | 1.846↓ | 7.695↓ | 0.447↓ | 14.809↓ | 0.24 ↓ |
| Station *2* | | 2.769↓ | 5.051↓ | 0.799↓ | 25.886↓ | 0.29 ↓ |
| Station *3* | | 4.651↓ | 2.814↓ | 0.619↓ | 29.733↓ | 0.13 ↓ |
| Center *0* | | 0.934↓ | 14.379↓ | 0.116↓ | 15.75↓ | 0.12 ↓ |

| | | | | | | |
|---|---|---|---|---|---|---|
| Station *1* | $K_i$×10 | 2.134↑ | 11.109↑ | 0.274↓ | 20.113↑ | 0.13 ↓ |
| Station *2* | | 3.139↑ | 7.402↑ | 0.482↓ | 35.188↑ | 0.15 ↓ |
| Station *3* | | 5.598↑ | 4.215↑ | 0.425↓ | 41.343↑ | 0.08 ↓ |
| Center *0* | | 1.144↑ | 21.474↑ | 0.075↓ | 21.474↑ | 0.07 ↓ |
| Station *1* | $K_i$×0.1 | 2.24↑ | 6.555↓ | 1.174↑ | 13.676↓ | 0.52 ↑ |
| Station *2* | | 3.548↑ | 4.352↓ | 2.112↑ | 24.872↓ | 0.6 ↑ |
| Station *3* | | 4.837↓ | 2.533↓ | 1.323↑ | 31.873↓ | 0.27 ↑ |
| Center *0* | | 0.949↓ | 14.193↓ | 0.237↑ | 14.193↓ | 0.25 ↑ |

- The change of the parameters influence on the maximal stock level $Q_i$ ($t1$): The change of the maximal stock level depends on that of parameters $CO_i$, $Cb_i$, $CP_i$ and $K_i$. They have the same change direction. As ordering cost $CO_i$ gets larger (smaller), the maximal stock level becomes larger (smaller).
- The change of the maximal stock level depends on that of parameters $h_i$ and $P_i$. But they have opposite change directions. As deterioration cost $P_i$ gets larger (smaller), the maximal stock level becomes smaller (larger).
- The change of the parameters influence on the periodic production time $t1^*$: The change of the periodic production time $t1^*$ depends on that of parameters $CO_i$, $Cb_i$, $CP_i$, $h_i$, $K_i$ and $P_i$. The change directions of $t1^*$ and $CO_i$, $Cb_i$, $CP_i$ are the same, but opposite between $t1^*$ and $h_i$, $K_i$, $P_i$.
- The change of the parameters influence on the producing/delivering proportion $t1^*/t2^*$: The change of the producing/ delivering proportion $t1^*/t2^*$ depends on that of parameters $CO_i$, $Cb_i$, $CP_i$, $h_i$, $K_i$ and $P_i$. The change directions of $t1^*/t2^*$ and $CO_i$, $Cb_i$, $CP_i$ are the same, but opposite between $t1^*/t2^*$ and $h_i$, $K_i$, $P_i$. Under the comparison of producing/ delivering proportion and the parameters described above, the producing/delivering proportion is relatively sensitive for the production rate (or processing/packaging rate). Therefore, stability control of the production rate (or processing/ packaging rate) is very helpful in suggesting a set of advisable extensive SCN strategies. The maximal stock level is indirectly controlled by the application of these factors, and they will be very helpful in indirect effect of various SCN activities and resources of the enterprises.

# 8. Conclusion

The production/distribution of the products and management of the stock quantity are worthy of our continued attention. This paper investigates time-based models for SCNs that are appropriate when an organization is faced with time-dependent linear demand and want to maintain an inventory of the item most of the time. The inventory decreases under deterioration (vaporization, damage, spoilage, etc.) during their normal storage period. As a result, while determining the optimal inventory policy of different types of product, the loss due to deterioration should not be neglected. The complication is more than general inventory management. In the course of study on two-echelon SCN systems with time-dependent linear demand, this paper proposed some conclusions as following:

- The objective of this paper is to solve the problem of the producing/ delivering management in SCN system by applying calculus, and to simplify regression analysis for the manager to use efficiently. Its key factors are the partition of time state and demand quantity obtaining. They have been proved and skills are offered in sections 5 and 6. Its advantages are using simple matrix calculation and modeling process. Through the linked information technology with this paper's viewpoints and models, we find that to obtain production period, stationary production/distribution, order quantity and average total cost, the manager only needs to enter the time-varying inventory levels when parameter dates are not changed. This paper's viewpoints and models also suit for SCN system using other random demand items. They can also be extended to management system using multi-echelon and multi-merchandise.
- This research uses the view of the total average unit cost to explain the decision behavior of SCN and basis of the control strategy. From the result of the numerical examples, we can make decision for the production/distribution center. If the center desire to reduce the possibility of the stock happened, but due to the emergence of production from the downstream business/ distribution stations, it would have to use phase-out production and deliver the goods promptly as often as possible, except the goods need to be processed. Accordingly, consider to regard for integrated cost of the industry-wide SCN with steady delivery, phase-out production, replenishment promptly and

the accurate goods are ordered in real-time etc., will make industry-wide development sounder.

- This research, which proposes mathematical algorithm that obtains the result, has already proved that it is superior to Goswami and Chaudhuri's algorithm, and the optimal strategic planning and total average unit costs can be obtained.

The following viewpoints are some future research directions to assist the development of SCNs:

— The optimal architecture of a supply chain taken into account of the integration of the supply chain for speed and flexibility as well as cost reductions.

— The integration of a virtual supply chain (network of partners) using ERP.

— Performance measures and matrices for multi-echelon availability models in SCNs.

## Acknowledgments

The author would like to thank the Editors-in-Chief for their comments and suggestions.

## References

[1] Abad, P.L., Jaggi, C.K., (2003) A joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. International Journal of Production Economics, 83, 115-122.

[2] Bhagwat, R. and Sharma M.K., (2007) Performance measurement of supply chain management: A balanced scorecard approach. Computers & Industrial Engineering, 53, 43–62.

[3] Chen, F. and Samroengraja, R., (2000) A staggered ordering policy for one-warehouse, multi-retailer systems. Operations Research, 48(2), 281-293.

[4] Chen, F.R., Fenergruen, A., Zheng, Y.S., (2001) Near-optimal pricing and replenishment strategies for a retail/distribution system, Operations Research, 49(6), 839-853.

[5] Christopher, M., (1992) Logistics and supply chain management. London: Pitman Publishing.

[6] Dale, B.G., Lascelles, D.M., Lloyd, A., (1994) Supply chain management and development. In Dale, B.G. (Eds). Managing Quality.

Prentice-Hall, London, pp.292-315.

[7] Dong, J. F., Du, S. F., Yang, S., Liang, L., (2008) "Competitive Pricing and Replenishment Policies in Distributed Supply Chain for a Deteriorating Item: A Game Approach," Asia Pacific Management Review, 13(2), pp. 497-512.

[8] Dye, C.Y., Ouyang, L.Y., Hsieh, T.P., (2007) Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. European Journal of Operational Research, 178(3), 789-807.

[9] Du, S.F., Liang, L., Yang, J., Qiu, H., (2006) Hybrid replenishment and dispatching policy with deteriorating items for VMI: analytical model and OSJCA approach. Asia Pacific Management Review, 11(3), 177-185.

[10] Feng Y, Cai XQ, Tu FS., (2008). Deteriorating inventory model under two-echelon integrated supply chain environment. Computer Integrated Manufacturing 14(2):300-305.

[11] Feng, Y. and Xiao, B., (2006) Integration of pricing and capacity allocation for perishable products. *European Journal of Operational* Research, 168(1), 17-34.

[12] Ganeshan, R., (1999) Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model. International *Journal of Production Economics*, 59(1-3), 341-354.

[13] Goswami, A. and Chaudhuri K., (1991) "EOQ model for an inventory with a linear trend in demand and finite rate of replenishment considering shortages," INT. J. Systems SCI., 22, No.1, pp.181-187.

[14] Gunasekaran, A., Yusuf, YY., (2002) Agile manufacturing: the taxonomy of strategic and technological imperatives, International Journal of Production Research 40(6):357–85.

[15] Huang, JY., Yao, MJ., (2005) On optimally coordinating inventory for a deteriorating item in a supply chain system with a single vendor and multiple buyers. Journal of the Chinese Institute of Industrial Engineers 22(6):473-484.

[16] Jarillo, J.C., (1993) Strategic Networks: Creating the Borderless Organization. Butterworth Heine-mann, Oxford.

[17] Lee, C.W., (2008) Green suppliers with environmental performance in the supply chain perspective. Asia Pacific Management Review, 13(4), 731-745.

[18] Lin C., Lin Y., (2007) A cooperative inventory policy with deteriorating items for a two-echelon model. European Journal of Operational Research 178(1):92-111.

[19] Mabert, V.A., Venkataramanan, M.A., (1998) Special research focus on supply chain linkages: Challenges for design and management in the 21st century. Decision Science, 29(3), 537- 552.

[20] Manna, S.K., Chaudhuri, K.S., (2006) An EOQ model with ramp type demand rate, time dependent deterioration rate, unit production cost and shortages. European Journal of Operational Research, 171, 557-566.

[21] McGill, J.I. and Ryzin, G.J. (1999) Revenue management: research overview and prospects. *Transportation Science*, 33(2), 233-256.

[22] Naylor, JB., Naim, MM., Berry, D., (1999) Leagility: integrating the lean and agile manufacturing paradigms in the total supply chain. International Journal of Production Economics 62, 107–18.

[23] Stadtler, H., (2000) Supply chain management: An overview. Stadtler, H. and Kilger, C. (ed.), Supply Chain Management and Advanced Planning: Concepts, Models, Software and Case Studies. Berlin, pp.7-29.

[24] Stevens, G.., (1989) Integrating the supply chain. International Journal of Physical Distribution and Materials Management, 19(8), 3-8.

[25] Viswanathan, S., Piplani, R., (2001) Coordinating supply chain inventories through common replenishment epochs. European Journal of Operational Research, 129(2), 277-286.

[26] Yang, P.C., Wee, H.M., (2003) An integrated multi-lot-size production inventory model for deteriorating item. Computers and Operations Research, 30, 671-682.

[27] Yoon, M. G., (2007) Capacity Control and Distribution problem for Manufactures in Supply Chain Networks. Asia Pacific Management Review 12(5), 26

[28] Youssef, MA., (1994) Agile manufacturing: the battleground for competition in the 1990s and beyond, International Journal of Operations Production Management 14(11):4–6.